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RETAINING WALLS
FOR EARTH

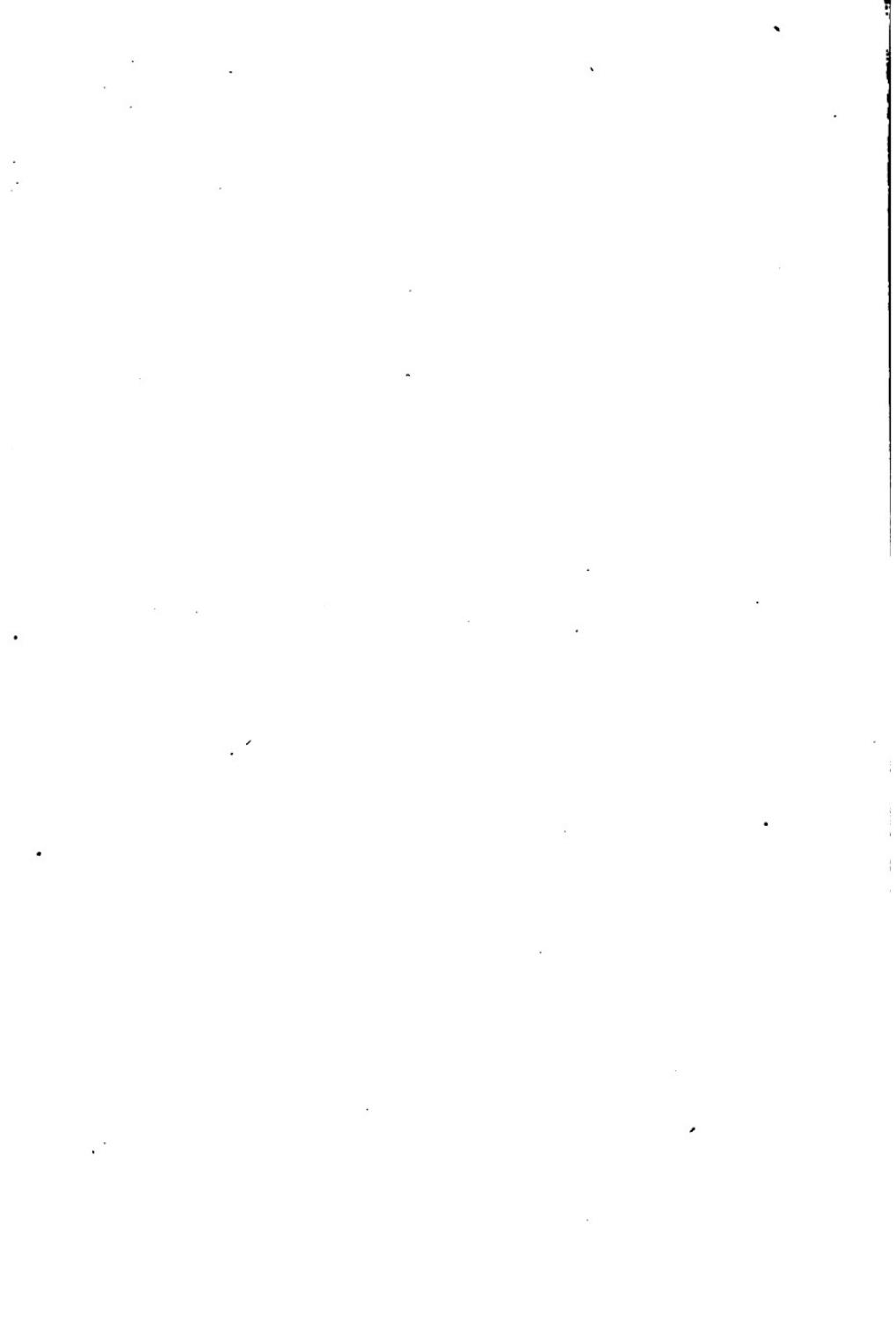
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RETAINING-WALLS FOR EARTH.

INCLUDING

*THE THEORY OF EARTH-PRESSURE
AS DEVELOPED FROM THE
ELLIPSE OF STRESS.*

WITH

A SHORT TREATISE ON FOUNDATIONS, ILLUSTRATED
WITH EXAMPLES FROM PRACTICE.

BY

MALVERD A. HOWE, C.E.,
*Professor of Civil Engineering, Rose Polytechnic Institute;
Member American Society of Civil Engineers.*

FIFTH EDITION, REVISED AND ENLARGED.

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PREFACE TO THE SECOND EDITION.

THE first edition of this work was based upon the theory advanced by Prof. Weyrauch in 1878, but owing to the length of the demonstrations used by him, it was thought advisable to present different and shorter demonstrations in this edition. To show that the new demonstrations give identical results with those obtained by Prof. Weyrauch, his demonstrations have been given in an appendix as they appeared in the first edition.

The new demonstrations are based upon the theory first advanced by Prof. Rankine in 1858. Those readers who are familiar with Rankine's Ellipse of Stress can omit pages 1 to 9, inclusive, in following the demonstrations.

An attempt has been made to present the theory in a shape easily followed by those who have only a knowledge of algebra, geometry, and trigonometry; whenever calculus has been resorted to, the work has been simplified as much as possible. For convenience in practice, the formulas have been arranged in a condensed shape in Part I, and are followed by numerous examples illustrating their application.

The values of various coefficients have been computed and tabulated and will be found to very materially decrease the labor of substitution in the formulas.

It is hoped that the introduction of a brief treatment of the supporting power of earth in the case of foundations, as well as the formula for determining the breadth of the base of a retaining-wall, will prove acceptable.

For valuable help in the verification of proofs of formulas, and the critical reading of the whole text, I acknowledge the kind assistance of Prof. Thos. Gray.

M. A. H.

PREFACE TO THE THIRD EDITION.

IN this edition a large number of examples illustrating several profiles of retaining-walls and types of foundations selected from existing structures have been included. The Appendix of the second edition has been replaced by a treatise on Foundations sufficiently short and, the author believes, sufficiently complete for the use of technical schools, if judiciously supplemented by lectures or references to descriptions of existing structures.

M. A. H.

TERRE HAUTE, IND., Nov. 1896.

PREFACE TO THE FOURTH EDITION.

IN this edition the few errors found in the previous edition have been corrected. Table I has been considerably enlarged. Reinforced-concrete retaining-walls have been considered in an Appendix of thirty pages, which includes the solution, in detail, of two examples. The formulas on pages 140-143 are presented through the courtesy of Mr. Edwin Thacher, who verified their correctness in the proof-sheets.

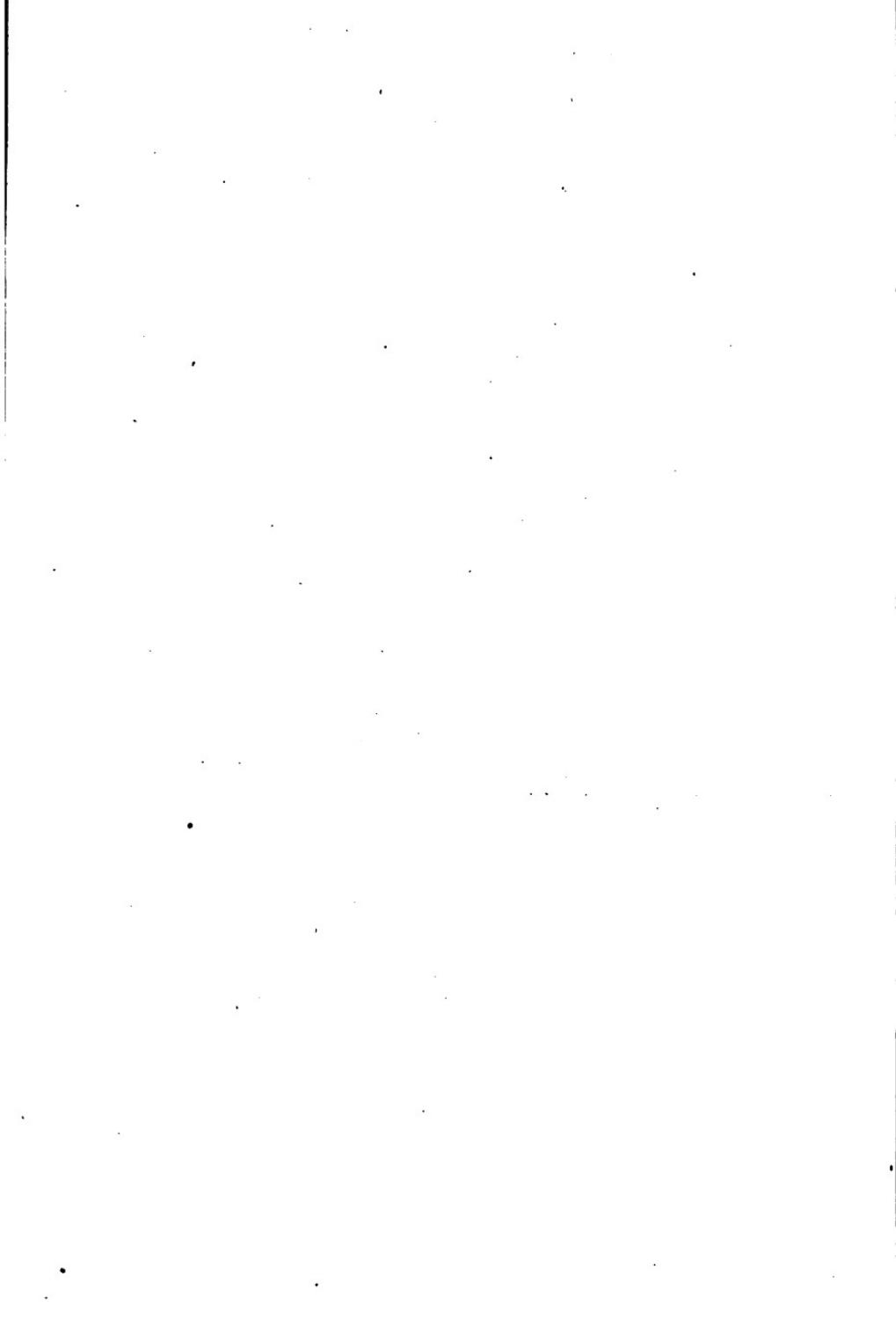
M. A. H.

PREFACE TO THE FIFTH EDITION.

A NUMBER of changes have been made in this edition. The formulas for offsets in foundations have been revised. Appendix A, which considers reinforced concrete retaining walls, has been rewritten to conform with standard nomenclature and formulas. A large number of profiles of walls actually built are shown in Appendices A and B.

M. A. H.

TERRE HAUTE, IND., June, 1911.



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NOMENCLATURE.

- ϕ = the angle of repose, or the maximum angle which any force acting upon any plane within the mass of earth can make with the normal to the plane.
- ϵ = the angle made by the surface of the earth with the horizontal; ϵ is *positive* when measured *above* and *negative* when measured *below* the horizontal.
- α = the angle which the back of the wall makes with the vertical passing through the heel of the wall; α is *positive* when measured on the *left* and *negative* when measured on the *right* of the vertical.
- δ = the angle which the direction of the resultant earth-pressure makes with the horizontal.
- ϕ' = the angle of friction between the wall and its foundation.
- ϕ'' = the angle of friction between the back of the wall and the earth.
- H = the vertical height of the wall in feet.
- h = the depth of earth in feet which is equivalent to a given load placed upon the surface of the earth.
- B' = the width in feet of the top of the wall.
- B = the width in feet of the base of the wall.
- Q = the distance in feet from the toe of the wall to the point where R cuts the base.

P = the resultant earth-pressure in pounds against a vertical wall.

E = the resultant earth-pressure in pounds against any wall.

R = the resultant pressure in pounds on the base of the wall.

G = the total weight in pounds of material in the wall.

γ = the weight in pounds of a cubic foot of earth.

W = the weight in pounds of a cubic foot of wall.

p = the intensity of the pressure in pounds on the base of the wall at the toe.

p' = the intensity of the pressure in pounds on the base of the wall at the heel.

p_a = the average intensity of the pressure in pounds on the base of the wall.

$x = H \tan \alpha$.

x'' and x' = depth of the base of the foundation below the earth surface.

B'' = breadth of the base of the foundation.

o = the offset of a foundation course.

G' = the total weight of the material above the base of the foundation.

THEORY OF EARTH-PRESSURE.

Preliminary Principles.—Before demonstrating the general formula for the thrust of earth against a wall, it will be necessary to establish the relations between the stresses in an unconfined and homogeneous granular mass.

* In Fig. 1 let AB be any small prism within a granu-

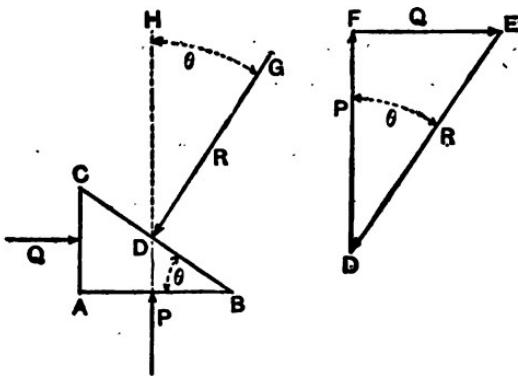


FIG. 1.

lar mass which is in equilibrium under the action of the three stresses P , Q , and R , having the intensities p , q , and r respectively.

* In all the demonstrations which follow, the dimension perpendicular to the page will be considered as unity.

THEORY OF EARTH-PRESSURE.

Let θ represent the angle of inclination of the plane CB with AB , and the angle at A be a right angle.

The planes AB and AC are called planes of principal stress, and P and Q are called principal stresses.

CASE I. If the principal stresses are of the same kind and their intensities the same, then will the resultant stress on any third plane be normal to that plane and its intensity be equal to that of either principal stress.

In Fig. 1, for convenience, let $AB = 1$, then $AC = \tan \theta$, and $CB = \frac{1}{\cos \theta}$. Hence

$$P = p, Q = q \tan \theta = p \tan \theta, \text{ since } p = q, \text{ and } R = \frac{r}{\cos \theta}.$$

Since P , Q , and R are in equilibrium, they will form a closed triangle, as shown on the right in Fig. 1. Hence

$$R^2 = P^2 + Q^2,$$

or

$$\frac{r^2}{\cos^2 \theta} = p^2 + p^2 \tan^2 \theta = p^2(1 + \tan^2 \theta);$$

$$\therefore r = p = q.$$

Also,

$$R \cos FDE = P,$$

or

$$\frac{r}{\cos \theta} \cos FDE = p; \text{ but } r = p.$$

Hence

$$\cos \theta = \cos FDE = \cos HDG;$$

$$\therefore HDG = \theta \text{ and } R \text{ is normal to } CB.$$

CASE II. If the principal stresses are not of the same kind but their intensities the same, then will the resultant make the angle θ with the direction of the principal stress, but on the opposite side from that on which the resultant in Case I lies, and its intensity be equal to that of either principal stress.

The demonstration of Case I proves this principle if Fig. 1 is replaced by Fig. 2.

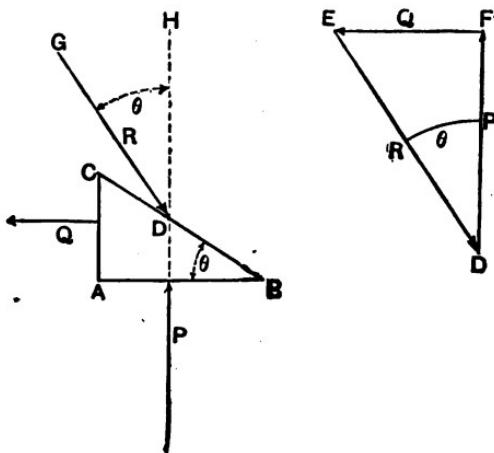


FIG. 2.

CASE III. Given the principal stresses of the same kind but having unequal intensities, to determine the intensity and direction of the resultant stress on any third plane.

Let P and Q be compressive and the intensity $p >$ the intensity q .

The following identities can be written:

$$p = \frac{1}{2}(p + q) + \frac{1}{2}(p - q),$$

and

$$q = \frac{1}{2}(p + q) - \frac{1}{2}(p - q);$$

or the resultant intensity on the plane CB may be considered as being the resultant of two intensities, one being the intensity of the resultant stress caused by two like principal stresses having the same intensity $\frac{1}{2}(p + q)$, and the other the intensity of the resultant stress caused by two unlike principal stresses having the same intensity $\frac{1}{2}(p - q)$.

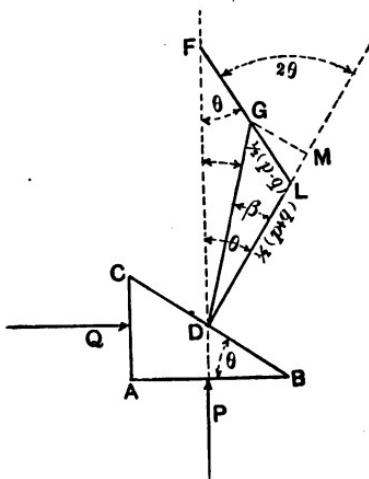


FIG. 3.

The intensity of the resultant stress caused by the first two principal stresses will be, by Case I, $\frac{1}{2}(p + q)$, and the direction of the resultant will be normal to the plane CB . By Case II the resultant of the second pair of principal stresses will make the angle θ with the direction of P , and its intensity will be $\frac{1}{2}(p - q)$; then the resultant intensity can be found as follows:

In Fig. 3 draw MD normal to BC , and make $LD = \frac{1}{2}(p + q)$; with L as a centre and LD as radius, describe an arc cutting FD at F . Then the angle $LFD = LDF = \theta$. Lay off $LG = \frac{1}{2}(p - q)$, and draw GD , which is the result-

ant intensity, and the intensity of the resultant stress on CD caused by the two principal stresses P and Q . GD also represents the direction of the resultant stress R .

Since the intensities of the principal stresses remain constant, $\frac{1}{2}(p+q)$ and $\frac{1}{2}(p-q)$ will remain the same for any inclination of the plane CB ; hence the intensity r of the resultant depends upon the angle θ when p and q are given.

From Fig. 3,

$$GL \cos 2\theta = LM \quad \text{and} \quad GL \sin 2\theta = GM,$$

$$DM = DL + LM = \frac{1}{2}(p+q) + \frac{1}{2}(p-q) \cos 2\theta,$$

$$\overline{GD}^2 = r^2 = \overline{GM}^2 + \overline{DM}^2,$$

or

$$r = \sqrt{p^2 \cos^2 \theta + q^2 \sin^2 \theta}, \dots \dots \quad (a)$$

which is the general expression for the intensity of the resultant stress of a pair of principal stresses.

As the angle θ changes, the angle β will also change, and it will have its maximum value when the angle $LGD = 90^\circ$. This is easily proven as follows:

With L as centre and GL as radius describe an arc; then β will have its maximum value when the line DG is tangent to the arc; but when DG is tangent to the arc the angle LGD is a right angle, since LG is the radius of the arc.

$$\sin \max \beta = \frac{p - q}{p + q}, \dots \dots \quad (b)$$

from which the following can be easily obtained:

$$\frac{p}{q} = \frac{1 + \sin \max \beta}{1 - \sin \max \beta}, \dots \dots \quad (c)$$

which expresses the limiting ratio of the intensities of the principal stresses consistent with equilibrium, p being greater than q .

CASE IV. Given the intensity and direction of the resultant stress on any plane, and the value of max β , to determine the intensities and directions of the principal stresses.

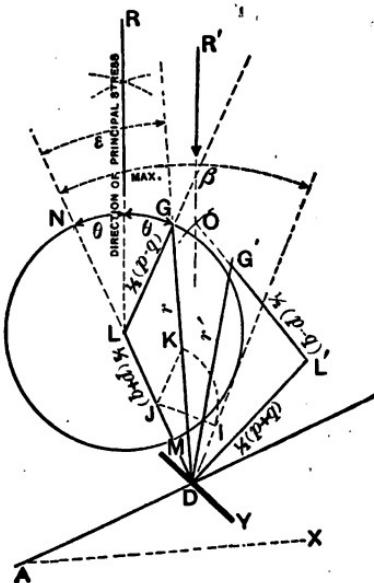


FIG. 4.

Let AD represent the given plane and GD the direction and intensity of the resultant stress at the point D .

Draw DL normal to AD , and draw DI , making the angle $\text{max } \beta$ with LD . At any point J in DL describe an arc tangent to DI , cutting GD in K and draw GL parallel to KJ ; with L as a centre and LG as radius describe

a circumference. This circumference will pass through G and be tangent to DI ; hence $\frac{GL}{DL} = \sin \max \beta$.

Since $\sin \max \beta = \frac{p - q}{p + q}$, and GL and LD are components of r ,

$$GL = \frac{1}{2}(p - q) \quad \text{and} \quad DL = \frac{1}{2}(p + q);$$

$$\text{then } ND = NL + LD = \frac{1}{2}(p - q) + \frac{1}{2}(p + q) = p,$$

$$\text{and } MD = LD - LM = \frac{1}{2}(p + q) - \frac{1}{2}(p - q) = q,$$

which completely determines the intensities of the principal stresses.

According to Case III, the direction of the greater principal stress bisects the angle between the prolongation of LM and the line GL ; hence RL represents the direction of the greater principal stress, and that of the other is at right angles to RL .

The above intensities and directions being determined, the intensity of the resultant stress on any other plane passing through D is easily determined as follows:

Let DY represent any plane passing through D , draw DL' normal to DY and equal to $\frac{1}{2}(p + q)$. Draw $R'D$ parallel to RL , and with L' as a centre and $L'D$ as radius describe an arc cutting $R'D$ at O , and make $L'G' = \frac{1}{2}(p - q)$; then $G'D = r' =$ the intensity of the resultant stress on DY .

It is clear that if the value of $\max \beta$ can be obtained for a mass of earth that the construction of Fig. 3 can be employed in determining the intensity of the earth-pressure at any point in *any plane* within the mass.

It has been established by experiment that if a body be placed upon a plane, that (as the plane is made to incline to the horizontal) at some angle of inclination the body will commence to slide down the plane, and that this angle depends largely upon the *character* of the surfaces in contact.

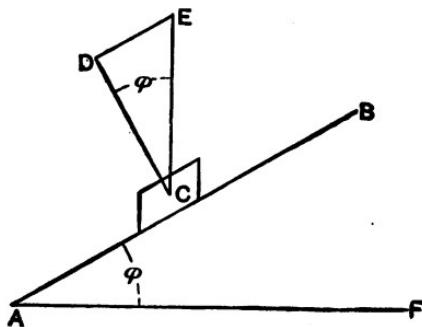


FIG. 5.

In Fig. 5 let AB represent a plane inclined at the angle ϕ with the horizontal, and C any mass just on the point of sliding down the plane. Let EC represent the weight of the mass C , and ED and DC the components respectively parallel and normal to the plane AB . Then DE is the force required to just keep the mass C from sliding down the plane, assuming the plane to be perfectly smooth, or if the plane is rough this force represents the effect of friction.

$$\frac{DE}{DC} = \tan \phi,$$

or when the mass C is about to slide, the resultant pressure EC on AB makes the angle ϕ with the normal to the

plane, the angle ϕ being the inclination of the plane AB , and is called the angle of friction.

In the case of earth, considered as a dry granular mass, the inclination of the steepest plane upon which earth will not slide is called the angle of repose, and the plane the surface of repose.

From the above, then, it follows that in a mass of earth the resultant pressure on any plane cannot make an angle with the normal to that plane which is greater than the angle of repose ϕ ; therefore the construction of Case IV applies to earth when $\max \beta$ is replaced by ϕ . The values of ϕ for earth under various conditions are given in Table II.

The preceding principles will now be applied in determining the thrust of earth against a retaining-wall.

EARTH-PRESSURE.

In order that the formulas may not become too complex for practical use, it will be assumed that the earth is a homogeneous granular mass without cohesion. The surface of the earth will be considered to be a plane, and the length of the mass measured normally to the page as unity.

* Given the intensity and direction of the resultant stress at any point in any plane parallel to the surface of the earth, the inclination of the surface of the earth with the horizontal, and the angle of repose, to determine the intensity and direction of the resultant stress on a vertical plane passing through the same point.

* For comparison, see the "Technic," 1888; a construction by Prof. Greene.

The construction follows (see Fig. 4, above) directly from Rankine's Ellipse of Stress.

In Fig. 6 let BQ represent the surface of the earth, and D any point in the plane AD parallel to BQ ; draw DQ normal to AD ; and make the vertical GD equal to QD ; then $GD \cdot \gamma$ is the intensity of the resultant pressure at D . Draw DM , making the angle ϕ with LD , and with L as centre describe an arc tangent to DM and passing through G ; then by Case IV $LG \cdot \gamma = \frac{1}{2}(p - q)$, $LD \cdot \gamma = \frac{1}{2}(p + q)$,

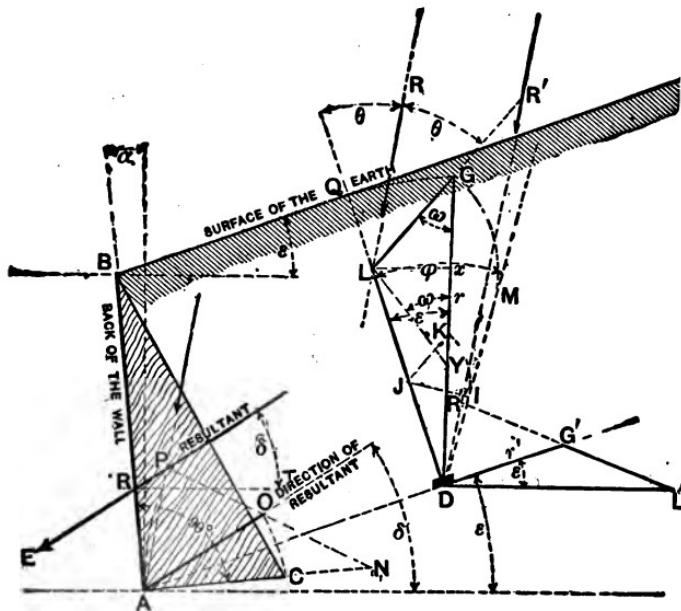


FIG. 6.

and RL bisecting the angle QLG is the direction of the greater principal stress. To determine the intensity and direction of the resultant stress at D on a vertical plane, proceed according to Case IV. Draw $R'D$ parallel to RL and $DL' = DL$ normal to DG . With L' as a centre and $L'D$ as radius describe an arc cutting $R'D$ at R'' , and make

$L'G' = LG$; then DG' represents the direction of the resultant stress, and $DG' \cdot \gamma$ the intensity of the resultant.

In Fig. 6 the angle $R'DL' = DR''L' = 90^\circ - \omega + \theta$.
 $\therefore G'L'D = 2\omega - 2\theta$. But $2\theta = \omega + \epsilon$; hence $G'L'D = \omega - \epsilon$.

Draw $LY = LG$; then the angle $DLY = \omega - \epsilon$. \therefore Since $LD = DL'$ and $LY = LG = L'G'$, the triangle $G'L'D$ equals the triangle LYD and the angle $G'DL' = \epsilon$; or the direction of the resultant earth-pressure against a vertical plane is parallel to the surface of the earth.

From Fig. 6,

$$\frac{1}{2}(p - q) \cos \omega = GX \cdot \gamma,$$

$$\frac{1}{2}(p - q) \sin \omega = LX \cdot \gamma,$$

$$\frac{1}{2}(p + q) \cos \epsilon = DX \cdot \gamma.$$

Now $DY = DG' = DG - 2GX$,

or

$$\begin{aligned} DG' \cdot \gamma &= DG \cdot \gamma - (p - q) \cos \omega \\ &= \frac{1}{2}(p + q) \cos \epsilon - \frac{1}{2}(p - q) \cos \omega, \\ \frac{1}{2}(p + q) : \sin \omega &\quad :: \quad \frac{1}{2}(p - q) : \sin \epsilon, \end{aligned}$$

and

$$\sin \omega = \frac{p + q}{p - q} \sin \epsilon,$$

or

$$\cos \omega = \sqrt{1 - \left(\frac{p + q}{p - q}\right)^2 \sin^2 \epsilon} = \sqrt{\frac{(p - q)^2 - (p + q)^2 \sin^2 \epsilon}{(p - q)^2}},$$

and since $\frac{1}{2}(p + q) \sin \phi = \frac{1}{2}(p - q)$,

$$\cos \omega = \frac{1}{\sin \phi} \sqrt{\cos^2 \epsilon - \cos^2 \phi}.$$

Substituting this value for $\cos \omega$ in the equation for $DG' \cdot \gamma$, it becomes

$$DG' \cdot \gamma = \frac{1}{2}(p+q) \cos \epsilon - \frac{1}{2}(p-q) \frac{1}{\sin \phi} \sqrt{\cos^2 \epsilon - \cos^2 \phi},$$

or since

$$\frac{1}{\sin \phi} = \frac{p+q}{p-q},$$

$$DG' \cdot \gamma = \frac{1}{2}(p+q) \{ \cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi} \}.$$

In a similar manner,

$$DG \cdot \gamma = \frac{1}{2}(p+q) \{ \cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi} \},$$

and

$$\frac{DG'}{DG} = \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}},$$

hence

$$DG' = DG \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}.$$

Let x = the *vertical* distance between the two planes BQ and AD , then

$$DG = DQ = x \cos \epsilon.$$

$$\therefore DG' \cdot \gamma = (x) \gamma \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}},$$

which is the expression for the intensity of the resultant earth-pressure on a vertical plane at any depth x below the surface.

Let

$$* A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}. \quad \dots \quad (d)$$

* See Rankine's Applied Mechanics; Alexander's Applied Mechanics; Theories of Winkler and Mohr.

The average intensity of the resultant earth-pressure on a vertical plane of the length x will be

$$\left(\frac{x}{2}\right)\gamma_A,$$

and hence the total pressure will be

$$P = \frac{x^3}{2} \gamma A \dots \dots \dots \quad (e)$$

Since the intensities of the pressures are uniformly varying from the surface, and increasing as x increases, the application of the resultant thrust will be at a depth of $\frac{1}{2}x$ below the surface.

Considering the earth as an unconfined mass, the above formula is perfectly general and can be applied under all conditions, including the case when ϵ is negative.

The resultant stress on any plane as AB , Fig. 6, can be found by applying the principles of Case IV. Draw PA parallel to RL , make $AN = LD$ and $NO = LG$; then AO represents the direction of the resultant pressure on AB . Make $AC = AO$; then the area of the triangle ABC multiplied by γ is the total pressure on the plane AB , and this pressure is applied at $\frac{1}{3}AB$ below B .

In unconfined earth this construction is perfectly general and applies to *any plane*. It also applies equally well to curved profiles. An example illustrating the application of the method will be given in the *applications*. See pages 54 and 55.

The following graphical construction, Fig. 7, is more convenient than that of Fig. 6.

As before, let BE represent the surface of the earth, and

AD a plane parallel to the surface. At any point *D* in this plane, draw *DE* vertical and make $DF = DE$; draw *FG* horizontal and make the angle $HFD = \phi$.

With L as a centre, describe an arc passing through G and tangent to MF ; then with L as a centre and LF as

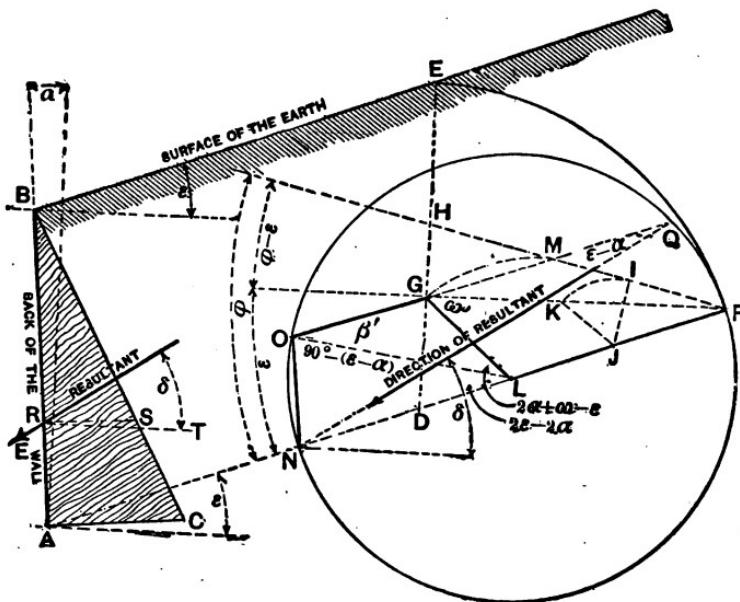


FIG. 7.

radius, describe the circumference FON , cutting AD at N ; through N draw NO parallel to AB , then draw AC normal to AB and equal to OG . The area of the triangle ABC multiplied by γ will be the total earth-pressure on AB . To determine the direction of the thrust prolong OG to Q , then QN is the direction of the thrust.

That this construction is equivalent to that of Fig. 6 is

proved as follows. The triangle GLF of Fig. 7 equals the triangle GLD of Fig. 6.

$$\therefore GL \cdot \gamma = \frac{1}{2}(p - q) \quad \text{and} \quad LF \cdot \gamma = LO \cdot \gamma = \frac{1}{2}(p + q).$$

In Fig. 6, the angle $NAP = NPA = 90^\circ - \frac{1}{2}(\omega - \epsilon) - \alpha$.

$$\therefore ONA = \omega - \epsilon + 2\alpha.$$

In Fig. 7, the angle $OLN = 2\epsilon - 2\alpha$. But $GLN = \omega + \epsilon$.

$$\therefore GLO = \omega - \epsilon + 2\alpha,$$

and GO of Fig. 7 equals AO of Fig. 6.

In Fig. 7, the angle $QNO = 90^\circ - \beta'$.

In Fig. 6, the angle $OAB = 90^\circ - \beta'$.

Therefore the direction of the thrust is the same in both constructions.

The two constructions given above are all that is required to determine the thrust of earth upon any plane within the mass of earth, as one can be used as a check upon the other; but as a formula is often very convenient, a general formula will now be deduced which will enable one to determine the values of E and δ for any plane within a mass of earth.

GENERAL FORMULA FOR THE THRUST OF EARTH.

In Fig. 8, let BQ represent the surface of the earth and AB any plane upon which the earth-pressure is desired.

Draw AD parallel to BQ and let the vertical distance $QD = FA = x$.

From (e) the earth-pressure upon FA is parallel to the surface and equal to

$$P = \frac{x^3}{2} \gamma A.$$

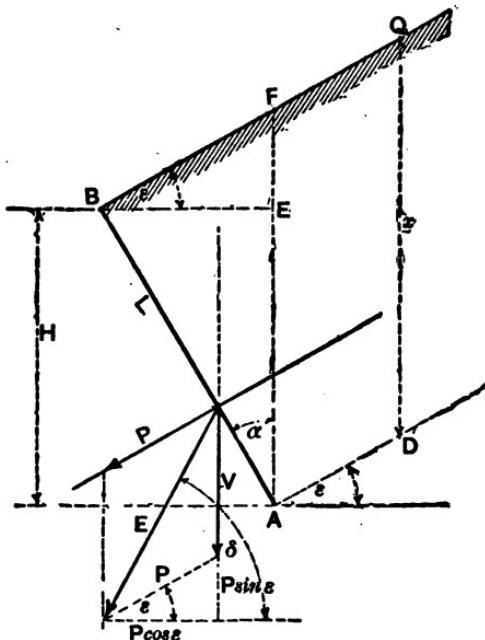


FIG. 8.

$$\text{But } AF = x = H(1 + \tan \alpha \tan \epsilon) = H \frac{\cos(\epsilon - \alpha)}{\cos \alpha \cos \epsilon};$$

$$\therefore P = \frac{H^3 \gamma}{2} \frac{\cos^3(\epsilon - \alpha)}{\cos^3 \alpha \cos^3 \epsilon} A. \quad \dots \quad (f)$$

Now the thrust P combined with the weight of the prism ABF must produce the resultant pressure upon AB .

Then from Fig. 8,

$$\begin{aligned} V &= \frac{H^2 \gamma}{2} \tan \alpha (1 + \tan \alpha \tan \epsilon) \\ &= \frac{H^2 \gamma}{2} \frac{\sin \alpha \cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon}, \quad (g) \end{aligned}$$

$$E = \sqrt{(V+P \sin \epsilon)^2 + (P \cos \epsilon)^2} = \sqrt{V^2 + P^2 + 2VP \sin \epsilon}.$$

Substituting (f) and (g) in this it becomes

$$E = \frac{H^2 \gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times \sqrt{\sin^2 \alpha + 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \frac{A}{\cos \epsilon} + \cos^2 (\epsilon - \alpha) \frac{A^2}{\cos^2 \epsilon}},$$

which becomes, by replacing A by its value from (d),

$$E = \frac{H^2 \gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times \sqrt{\begin{aligned} &+ \sin^2 \alpha \\ &+ 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \\ &+ \cos^2 (\epsilon - \alpha) \left\{ \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}^2 \end{aligned}}, \quad (1)$$

which is the general equation for the thrust of earth upon any plane within the mass.

To determine the direction of the thrust of the earth, let δ be the angle which the direction of the thrust makes with the horizontal; then, from Fig. 8,

$$\tan \delta = \frac{V}{P \cos \epsilon} + \tan \epsilon.$$

Substituting the values of V and P given above, this becomes

$$\tan \delta = \frac{\sin \alpha \cos \epsilon + \sin \epsilon \cos (\epsilon - \alpha) A}{\cos \epsilon \cos (\epsilon - \alpha) A}, \quad \dots \quad (1a)$$

where

$$A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}. \quad \dots \quad (d)$$

Equations (1) and (1a) are readily reduced to more simple forms for special cases. These forms will be found on pages 23-25.

The Plane of Rupture.—Although it is not necessary to know the position of the plane of rupture in order to determine the thrust of the earth, yet it may be of interest to know its position, which can be easily determined as follows:

The plane of rupture will be back of the wall and pass through the heel of the wall. The resultant earth-pressure will make the angle ϕ with the normal to this plane. Now the tangent of the angle which the direction of the resultant earth-pressure on any plane makes with the horizontal is determined from the formula

$$\tan \delta = \frac{\sin \alpha}{\cos (\epsilon - \alpha) A} + \tan \epsilon.$$

If ω represents the angle which the plane of rupture makes with the vertical passing through the heel of the wall, $\alpha = \omega$ and $\delta = \phi + \omega$.

$$\tan (\phi + \omega) = \frac{\sin \omega}{\cos (\epsilon - \omega) A} + \tan \epsilon,$$

from which the value of ω can be determined for any case.

For the case where $\epsilon = \phi$, ϵ being positive with respect to the wall and *negative with respect to the plane of rupture*, the above equation becomes

$$\tan(\phi + \omega) = \frac{\sin \omega}{\cos(\phi + \omega) \cos \phi} - \tan \phi,$$

which is satisfied when $\omega = 90^\circ - \phi$.

For the case where $\epsilon = 0$,

$$\tan(\phi + \omega) = \frac{\sin \omega}{\cos \omega \tan^2\left(45^\circ - \frac{\phi}{2}\right)},$$

which is satisfied when $\omega = 45^\circ - \frac{\phi}{2}$.

Reliability of the Preceding Theory.—The preceding theory is based upon the assumptions that the earth is a homogeneous mass and without cohesion, and the formulas are deduced under the assumption that the surface of the earth is a plane.

All writers on the subject have considered the earth as a homogeneous mass and, with a few exceptions, without cohesion.

Old and recent experiments indicate that cohesion has very little effect upon the pressure of the earth, which explains why it has not been considered by most writers.

The assumption of a plane earth-surface is necessary whenever practical formulas and direct graphical constructions for obtaining the thrust of the earth are obtained. General formulas can be deduced for any character of surface, but they are too complex for practical use. Those graphical constructions which do not require a plane earth-

surface are not direct in their solution of the problem, but require a series of trials to obtain the maximum thrust.

If the earth-surface is not a plane, one can be assumed which will give the thrust of the earth sufficiently exact for all practical purposes.

For unconfined earth no exceptions can be taken to the preceding theory, the assumptions upon which it is based being accepted, and for confined earth the theory must be true when the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

For all cases in which α and ϵ are positive the theories of *Rankine*, *Winkler*, *Weyrauch*, and *Mohr* agree and give identical results with the preceding theory, as they should, being founded upon the same assumptions.

When α is negative *Weyrauch* does not consider his theory reliable, and his equations lead to indeterminate results.

Winkler and *Mohr* consider their theories reliable whenever the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

Rankine's method of considering the case where α is negative is equivalent to assuming that the introduction of a wall does not affect the stresses within the mass.

It may be concluded that the preceding theory is perfectly exact when α and ϵ are positive; and when α or ϵ is negative that the stresses obtained will be the maximum which under any circumstances can exist.

For the case where ϵ is negative the stress obtained will be considerably larger than the actual stress (when a wall is introduced), depending upon the magnitude of ϵ . For small values of ϵ the results will be practically correct. For large values of ϵ the following method can be employed in determining the thrust of the earth. The

method depends upon the *assumption* that the pressure of the earth is normal to the back of the wall. This may or may not be the case, but it appears to be the most consistent assumption to make for this rare and not important case.

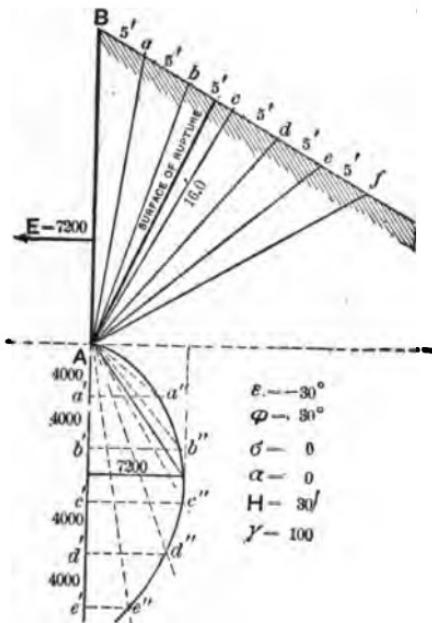


FIG. 9.

* In Fig. 9, let AB be the back of the wall and Bf the surface of the earth. Make $Ba = ab = bc = cd = \text{etc}$. Some prism BAa or BAb or BAc , etc., will produce the maximum thrust on the wall; and when this maximum thrust is produced, the resultant pressure on the plane Aa

* See Van Nostrand's Magazine, xvii, 1877, p. 5. "New Constructions in Graphical Statics," by H. T. Eddy, C.E., Ph.D.

or Ab or Ac , etc., will make the angle ϕ with the normal to the plane.

On the vertical line Ad' lay off $Aa' = a'b' = b'c'$, etc., and draw Aa'' making the angle ϕ with the normal to Aa , Ab'' making the angle ϕ with the normal to Ab , etc.; then draw $a'a'', b'b'', c'',$ etc., perpendicular to AB , and draw a curve through $Aa'', b'', c'',$ etc. Then there will be a maximum distance parallel to $a'a''$ between Ad' and this curve which will be proportional to the thrust of the earth against AB . This maximum distance multiplied by the altitude $Ac \div 2$ and the product by γ , the weight of a cubic foot of earth, will be the pressure of the earth.

This method is perfectly general and can be applied in any case.

If the earth-pressure is assumed to have the direction given by the formulas of the preceding theory, the construction will give the same value of E , the pressure of the earth.

Some writers assume that the direction of E makes the angle $\phi'' = \phi$ with the normal to the back of the wall in all cases. This assumption cannot be correct until the wall commences to tip forward, and then it is doubtful that such is the case unless the earth and wall are perfectly dry.

To be on the side of safety in every case, it is better to take the direction of E as given by the above theory.

The construction of Fig. 9 will give the maximum thrust for any assumed direction for any case.

FORMULAS FOR EARTH-PRESSURE.

IN the following formulas α and ϵ are considered as *positive*, and the wall is assumed to be one foot long.

CASE I. *General case of inclined earth-surface and inclined back of wall.*

$$E = \frac{H^2 \gamma}{2} \frac{\cos(\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times \sqrt{\sin^2 \alpha + \cos^2(\epsilon - \alpha) \left\{ \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}^2 + 2 \sin \epsilon \sin \alpha \cos(\epsilon - \alpha) \left\{ \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}}; \quad (1)$$

or

$$E = \frac{H^2 \gamma}{2} (B) \sqrt{(C) + (D)A^2 + (E)A}. \quad (1')$$

$$\tan \delta = \frac{\sin \alpha \cos \epsilon + \sin \epsilon \cos(\epsilon - \alpha)A}{\cos \epsilon \cos(\epsilon - \alpha)A}; \quad (1a)$$

$$\text{or } \tan \delta = \frac{\sin \alpha}{\cos(\epsilon - \alpha)A} + \tan \epsilon, \quad \quad (1'a)$$

where

$$A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \quad \dots \quad (d)$$

CASE II. Surface of earth inclined and $\alpha = 0$.

$$E = P = \frac{H^2 \gamma}{2} \left\{ \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} = A \right\}. \quad (2)$$

From Diagram I the values of A can be found for all values of ϕ from 0° to 90° and of ϵ from 0° to 90° , varying by 5° .

$$\delta = \epsilon; \dots \dots \dots \quad (2a)$$

or for all vertical walls the direction of the earth-pressure is parallel to the surface of the earth.

CASE III. The surface of the earth parallel to the surface of repose.

$$\epsilon = \phi.$$

$$E = \frac{H^2 \gamma}{2} \frac{\cos(\phi - \alpha)}{\cos^2 \alpha \cos \phi} \sqrt{\frac{\sin^2 \alpha + \cos^2(\phi - \alpha)}{+ 2 \sin \alpha \sin \phi \cos(\phi - \alpha)}}. \quad (3)$$

$$\tan \delta = \frac{\sin \alpha + \sin \phi \cos(\phi - \alpha)}{\cos \phi \cos(\phi - \alpha)}. \quad \dots \quad (3a)$$

CASE IV. The surface of the earth parallel to the surface of repose and the back of the wall vertical.

$$\epsilon = \phi \text{ and } \alpha = 0.$$

$$E = \frac{H^2 \gamma}{2} \cos \phi. \quad \dots \dots \dots \quad (4)$$

$$\delta = \phi. \quad \dots \dots \dots \quad (4a)$$

CASE V. *The surface of the earth horizontal.*

$$\epsilon = 0.$$

$$E = \frac{H^2 \gamma}{2} \sqrt{\tan^2 \alpha + \tan^4 \left(45^\circ - \frac{\phi}{2}\right)}. \quad . \quad (5)$$

$$\tan \delta = \frac{\tan \alpha}{\tan^2 \left(45^\circ - \frac{\phi}{2}\right)} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (5a)$$

CASE VI. *The surface of the earth horizontal and the back of the wall vertical.*

$$\epsilon = 0 \text{ and } \alpha = 0.$$

$$E = \frac{H^2 \gamma}{2} \tan^2 \left(45^\circ - \frac{\phi}{2}\right) \cdot \cdot \cdot \cdot \cdot \quad (6)$$

$$\delta = 0. \cdot \quad (6a)$$

CASE VII. *Fluid pressure.*

$$\epsilon = \phi = 0.$$

$$E = \frac{H^2 \gamma}{2 \cos \alpha} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (7)$$

$$\delta = \alpha. \cdot \quad (7a)$$

GRAPHICAL CONSTRUCTIONS FOR DETERMINING THE THRUST OF EARTH.

The following constructions are perfectly general, and apply to *any plane* within a mass of earth. When applied

for determining the thrust of earth against a *retaining-wall*, α and ϵ are taken as positive.

* *Construction (a).*

Let BE represent the surface of the earth and BA the back of the wall. Draw AF parallel to BE , and at any point D in AF lay off DF equal to the vertical DE . Draw

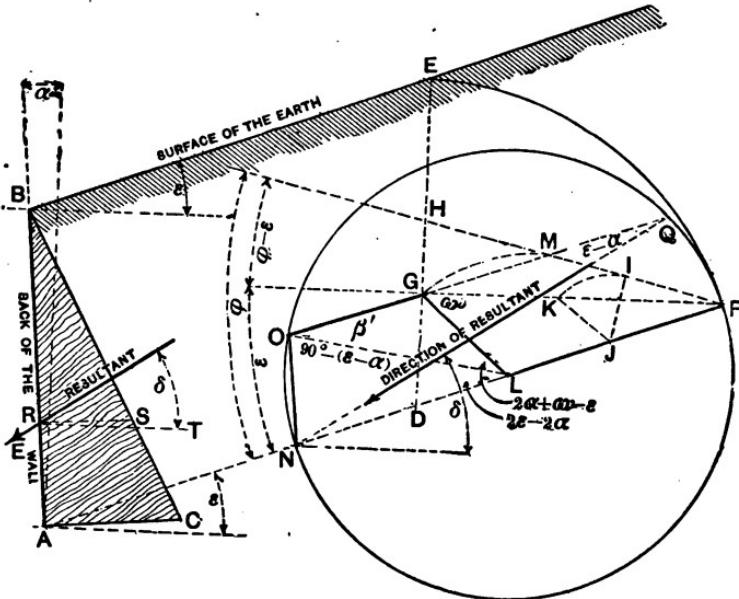


FIG. 10.

FG horizontal, and FH , making the angle ϕ with DF . With any point J in DF describe the arc KI tangent to HF at I cutting FG at K , and draw GL parallel to KJ ; with L as a centre and LF as radius, describe the circumference $FQON$ cutting AD at N . Through N draw NO

* See "Theorie des Erddruckes auf Grund der neueren Anschauungen," by Prof. Weyrauch, 1881.

parallel to AB cutting the circumference $FQOV$ at O ; at A draw AC equal to OG and normal to AB ; the area of the triangle ABC multiplied by γ will be the thrust of the earth on the wall.

To determine the direction of the thrust E , prolong OG to Q ; then QN will be the direction of the thrust.

This thrust acts on the wall at $\frac{2}{3}AB$ below B .

** Construction (b).*

Let BQ represent the surface of the earth, and BA the back of the wall. Draw AD parallel to BQ , and at any

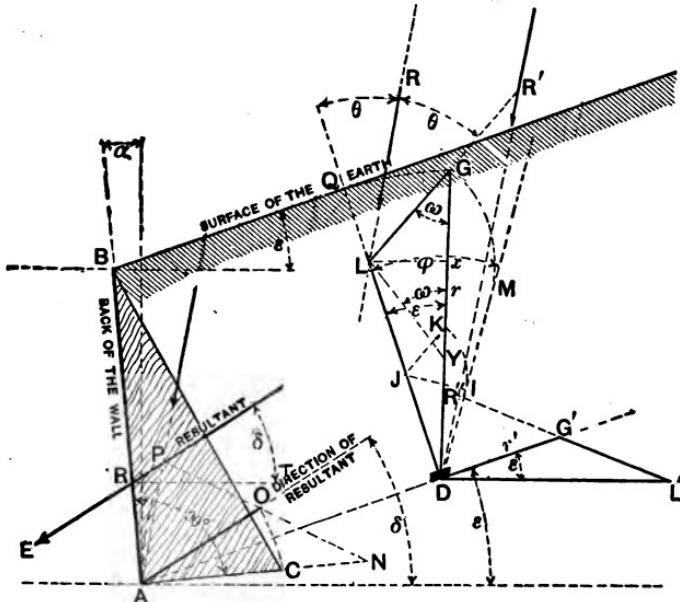


Fig. 11.

point D in AD draw the vertical DG equal to the normal DQ ; draw DM making the angle ϕ with the normal DQ .

* This construction follows directly from Rankine's Ellipse of Stress. See Rankine's Applied Mechanics.

At any point J in DQ as a centre, describe the arc IK tangent to DM cutting DG at K , and draw GL parallel to JK . Bisect the angle QLG , and at A draw AP parallel to LR . At A draw AN normal to AB and equal to DL ; with N as a centre and AN as radius, describe an arc AP cutting AP at P ; connect P and N , and make NO equal to LG ; with A as a centre and AO as a radius, describe the arc OC cutting AN at C ; then the area of the triangle ABC multiplied by γ will be the thrust against the wall. The direction of this thrust is parallel to AO and it is applied at $\frac{2}{3}AB$ below B .

The constructions (a) and (b) give identical results in every case.

STABILITY OF TRAPEZOIDAL WALLS

As the majority of walls retaining earth are trapezoidal in section, the stability of such walls alone will be considered. If other forms occur in practice they can be divided into trapezoidal sections with horizontal beds, and the stability of each considered, commencing with the upper section.

Walls having the rear faces in the form of steps can usually be considered as trapezoidal in section by replacing the stepped portion by a straight line which approximately bisects each step. If the front faces are stepped they can be treated in a similar manner.

In case the front face of the wall is curved in profile, the curve may be replaced by straight lines which are chords of the curve, thus dividing the section into as many trapezoids as there are chords.

It will be assumed that the direction and magnitude of the earth-pressure is known, that the position and extent of the back of the wall, and the width of the top are given,

to determine the width of the base for stability against overturning, sliding, and crushing of the material.

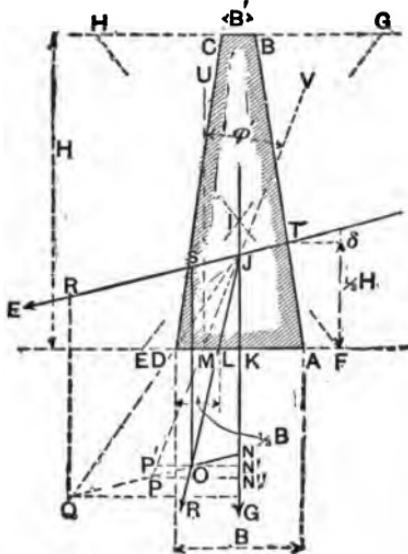


FIG. 12.

Stability against Overturning.—Let $ABCD$, Fig. 12, represent a section of a trapezoidal wall, TR the direction of the earth-thrust, JG the vertical passing through the centre of gravity of the wall, and JO the direction of the resultant pressure on the base AD caused by E and G .

As long as R cuts the base AD , the wall will be stable against overturning. When R takes the direction JQ , the wall may be said to be on the point of overturning; then the factor of safety against overturning is $\frac{QN}{ON}$, where ON is the actual value of E , and QN the value of E required to make the resultant R pass through D .

Stability against Sliding.—Since the wall will not slide

along the surface DA until the resultant R makes an angle with the normal to DA greater than the angle of friction ϕ' , the factor of safety against sliding can be obtained as follows: Draw JP making the angle $JMU = \phi'$; then the factor of safety against sliding is $\frac{PN}{ON}$, where PN is the force required in the direction of E to make R make the angle ϕ' with the normal to AD , and ON the actual value of E .

Stability against the Crushing of the Material.—In ordinary practice walls for retaining earth are not of sufficient height to cause very large pressures at their bases, but it is necessary to consider the subject on account of the tendency of the bed-joints to open under certain conditions.

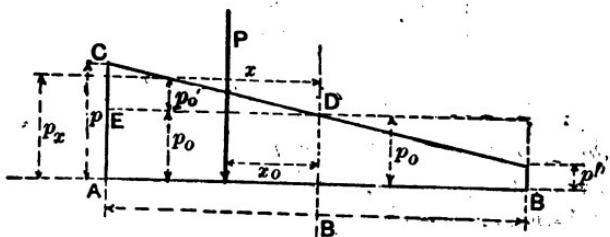


FIG. 13.

Let AB , Fig. 13, represent any bed-joint in the wall, P the vertical resultant pressure upon the joint, and x , the distance of the point of application from the centre of the joint.

The intensity of P at any point can be considered as composed of a uniform intensity $p_0 = \frac{P}{B}$, and a uniformly varying intensity p'_0 , so that $p_x = p_0 + p'_0$. Let a equal the tangent of the angle CDE , then $p'_0 = ax$ and $p_x = p_0 + ax$.

The pressure upon a surface (dx)—the joint being considered unity in the dimension normal to the page—is

$$p_a dx = p_o dx + ax dx,$$

and the moment of this about DB is

$$(p_o dx + ax dx)x.$$

The algebraic sum of these moments for values of x between the limits $\pm \frac{B}{2}$ must equal Px_o , or

$$Px_o = \int_{-\frac{B}{2}}^{+\frac{B}{2}} (p_o x dx + ax^2 dx).$$

Integrating,

$$a = \frac{12x_o P}{B^3} = \frac{12x_o p_o}{B^3},$$

and

$$p_a = \frac{B^3 + 12x_o x}{B^3} p_o,$$

or making $x = \frac{1}{2}B$,

$$p = \left\{ 1 + \frac{6x_o}{B} \right\} \frac{P}{B},$$

and if x_o be replaced by $\frac{1}{2}B - Q$, where Q is the distance from A to the point where P cuts the base, (Fig. 13),

$$p = 2 \left(2 - \frac{3Q}{B} \right) \frac{P}{B},$$

and

$$p' = 2 \left(-1 + \frac{3Q}{B} \right) \frac{P}{B}.$$

If $Q = \frac{1}{2}B$,

$$p' = 0 \quad \text{and} \quad p = 2p_o,$$

from which it is seen that when R cuts the base outside the middle third, the joint will have a tendency to open at points which are at a maximum distance from R where it cuts the base.

Therefore in no case should the resultant pressure be permitted to cut the base outside the middle third. This makes it unnecessary to consider the stability against overturning.

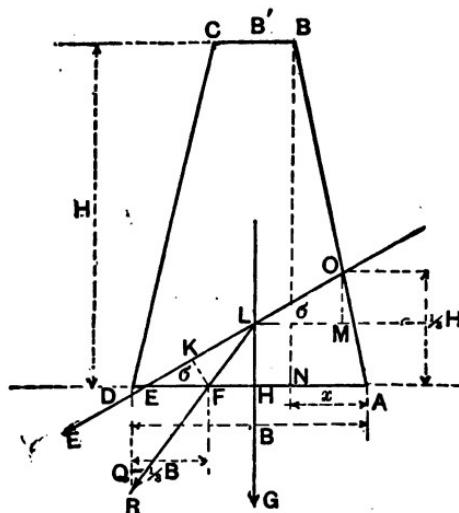


FIG. 14.

Then in designing a wall the following conditions must exist for stability:

- I. The resultant R must cut the base for stability against overturning.
 - II. The resultant R must not make an angle with the normal to the base of the wall greater than the angle of friction ϕ' .

III. The resultant R must not cut the base outside of the middle third, in order that there may be no tendency for the bed-joints to open.

The above three conditions apply to any bed-joint of the wall; but if they are satisfied at the base and the wall has the section shown in Fig. 14, it will not be necessary to consider any joints above the base unless the character of the stone or the bonding is different.

Determination of the width of the base of a retaining-wall under the condition that R cuts the base at a point $\frac{1}{3}B$ from the toe of the wall.

Let H , B' , x , δ , and E be given to determine B .

From Fig. 14,

$$KF = \frac{x}{3} \sin \delta + \frac{H}{3} \cos \delta - \frac{2B}{3} \sin \delta,$$

$$HD = \frac{2B^* + 2BB' - Bx - 2B'x - B''}{3(B + B')},$$

$$HF = HD - \frac{B}{3} = \frac{B^* + BB' - Bx - 2B'x - B''}{3(B + B')}.$$

For equilibrium

$$E(KF) = G(HF) = \frac{B + B'}{2} HW(HF).$$

Substituting the values of KF and HF in the above and reducing, it becomes

$$\begin{aligned} B^* + B\left(\frac{4E}{HW} \sin \delta + B' - x\right) \\ = \frac{2E}{HW}(H \cos \delta + x \sin \delta) + 2B'x + B'', \quad . \quad (8) \end{aligned}$$

which is the general equation for the width of the base of a trapezoidal wall.

For a rectangular wall $B' = B$.

For a triangular wall $B' = 0$.

For a wall with a vertical front $B' + x = B$ or $B' = B - x$.

For a wall with a vertical back $x = 0$.

Equation (8) is easily transformed to satisfy the requirements of special cases.

The width of the base can be found graphically by assuming a value for B and finding the value of Q ; if it is less than $\frac{1}{3}B$ another value of B must be assumed, and so on until Q is equal to or greater than $\frac{1}{3}B$.

FORMULAS FOR TRAPEZOIDAL AND TRIANGULAR WALLS.

Formulas for the width of the base of trapezoidal walls under the condition that the resultant R cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or $Q = \frac{1}{3}B$.

CASE I. *The general case in which the back of the wall is inclined, and E makes an angle with the horizontal.*

$$\begin{aligned} B'' + B \left(\frac{4E}{HW} \sin \delta + B' - x \right) \\ = \frac{2E}{HW} \left(H \cos \delta + x \sin \delta \right) + 2B'x + B'' . . \quad (8) \end{aligned}$$

CASE II. *The back of the wall vertical.*

$$x = 0.$$

$$B'' + B \left(\frac{4E}{HW} \sin \delta + B' \right) = \frac{2E}{W} \cos \delta + B'' . \quad (9)$$

CASE III. *The back of the wall vertical and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B^* + BB' = \frac{2E}{W} + B'^*. \quad \dots \quad (10)$$

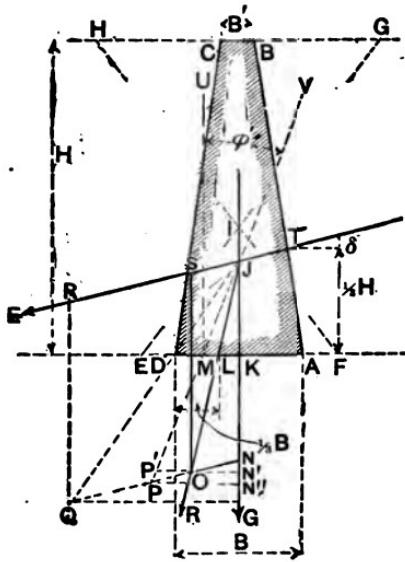


FIG. 15.

If $B = B'$ and $x = 0$, the section of the wall is a rectangle, and (9) becomes

$$B^* + B \frac{4E}{HW} \sin \delta = \frac{2E}{W} \cos \delta, \quad \dots \quad (9a)$$

and (10) becomes

$$B = \sqrt{\frac{2E}{W}}. \quad \dots \quad (10a)$$

Formulas for the width of the base of triangular walls under the condition that the resultant R cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or $Q = \frac{1}{3}B$.

CASE I. *The general case in which the back of the wall is inclined, and E makes an angle with the horizontal.*

$$B^* + B \left(\frac{4E}{HW} \sin \delta - x \right) = \frac{2E}{HW} (H \cos \delta + x \sin \delta). \quad (11)$$

CASE II. *The back of the wall vertical.*

$$\alpha = 0.$$

$$B^* + B \left(\frac{4E}{HW} \sin \delta \right) = \frac{2E}{W} \cos \delta. \quad . . . \quad (12)$$

CASE III. *The back of the wall vertical, and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B = \sqrt{\frac{2E}{W}}. \quad \quad (13)$$

The above formulas do not contain the condition that R shall not make an angle greater than ϕ' with the normal to the base of the wall.

From Fig. 15,

$$\tan \phi' \geq \frac{E \cos \delta}{G + E \sin \delta} = \tan LJK, \quad . . . \quad (14)$$

which expresses the condition under which the wall will not slide.

FOUNDATIONS FOR WALLS RETAINING EARTH.

The design of the foundations for retaining-walls has received but little attention by writers upon engineering subjects, and the practical engineer has not published to any great extent examples of the foundations he has employed under the countless number of walls erected along railways, highways, canals, etc.

As the designing of foundations resting upon earth, for walls retaining earth, introduces several features which do not influence the ordinary cases of foundations, it will be best to make a special investigation for such conditions.

The intensity of the foundation pressure upon the earth is seldom uniform, due principally to the pressure of the earth behind the wall and foundation tending to overturn the structure as a whole; this being the case, evidently the maximum intensity upon the earth must not be large enough to heave the earth, and the minimum intensity must not be so small that the earth may heave the foundation.

If the foundation be so designed that neither it nor the earth can be heaved, the structure may yet fail by sliding forward. This can only be resisted by the abutting power of the earth in front of the foundation and the friction upon the base of the foundation. Usually, however, if there is no danger of any movement in a vertical plane, there is little or no danger of any movement in a horizontal direction.

As in any structure good judgment must enter into the design, the formulas which will be demonstrated must be

used as guides only. These formulas will depend upon the angle of repose ϕ of a homogeneous granular mass, and the specific gravity of this mass. For ordinary earths for which the weights and angles of repose are known the results obtained by the use of the formulas will compare very favorably with those obtained from examples of the best practice.

Depth of Foundations.—Given the angle of repose ϕ of any earth, to determine the depth to which it is necessary to sink a foundation to support a given load. The surface of the earth is assumed to be horizontal.

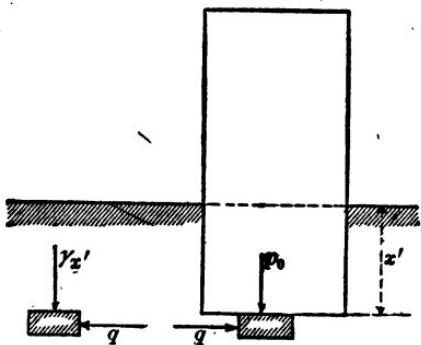


FIG. 16.

CASE I. When the intensity of the pressure on the base of the foundation is uniform.

In Fig. 16, let p_0 represent the intensity of the pressure on the base of the foundation.

Now when the masonry is about to sink (see Eq. (c)),

$$\frac{p_0}{q} = \frac{1 + \sin \phi}{1 - \sin \phi} \quad \text{or} \quad q = p_0 \cdot \frac{1 - \sin \phi}{1 + \sin \phi}$$

If x' represents the depth to which the foundation extends below the surface of the earth and γ the weight of a cubic

foot of earth, then $\gamma x'$ equals the vertical intensity of the earth-pressure on a plane at the depth of the lowest point of the foundation.

When the wall is on the point of sinking, the earth must be on the point of rising, or

$$\frac{P_o}{\gamma x'} = \frac{1 + \sin \phi}{1 - \sin \phi}$$

or

$$p_o = \gamma x' \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \dots \dots \quad (15)$$

In any case p_o must not have a greater value than that obtained from (15)—

$$x' = \frac{p_o}{\gamma} \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^{1/2} = \frac{p_o}{\gamma} \tan^2 \left(45^\circ - \frac{\phi}{2} \right). \quad (16)$$

The value of x' as obtained from (16) is the least allowable value consistent with equilibrium. Since x' is a function of $\tan^2 \left(45^\circ - \frac{\phi}{2} \right)$, care must be taken that ϕ is assumed at its least value. As ϕ becomes smaller the value of x' increases rapidly.

CASE II. *When the intensity of the pressure on the base is uniformly varying.*

Let p represent the maximum intensity of the pressure on the earth and p' the minimum intensity; then for equilibrium p must not exceed the value obtained from the following equation (see 15):

$$p = x' \gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \dots \dots \quad (17)$$

For any assumed depth x' the maximum value of p can be

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found from (17). For any assumed breadth B'' of the foundation the value of p due to the resultant pressure upon the base of the foundation can be found from the formulas on page 31, when the value of x_0 has been determined; this value must not be greater than the value of p found from (17), or the masonry will heave the earth.

In order that the earth may not heave the masonry, p' must not be *less* than the value obtained from the following formula:

$$p' = x' \gamma \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^* \dots \dots \quad (18)$$

Then

$$p_0 = \frac{p + p'}{2} = \frac{x' \gamma}{2} \left\{ \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)^* + \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^* \right\}, \quad (19)$$

which expresses the *corresponding* value of p_0 for the equilibrium of the earth and the masonry.

In order that p' may never be less than the value obtained from (18), the resultant pressure upon the base of the foundation must cut the base within a certain distance of its centre. If x_0 be this distance, then (page 31)

$$p' = x' \gamma \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^* = \left\{ 1 - \frac{6x_0}{B''} \right\} p_0. \quad (20)$$

Substituting the value of p_0 from (19) and solving for x_0 ,

$$x_0 = \frac{B''}{6} \left\{ \frac{X - Y}{X + Y} \right\}, \quad \dots \dots \quad (21)$$

where

$$* X = \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)^* \quad \text{and} \quad Y = \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^*.$$

* Tabulated values of X and Y are given on page 72.

Depth of foundations when the surface of the earth has different elevations on opposite sides of the structure.

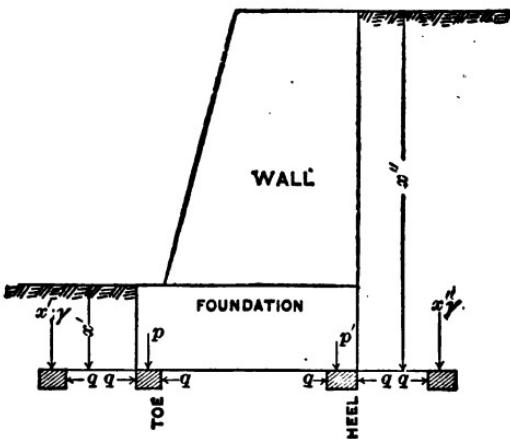


FIG. 17.

This case is illustrated in Fig. 17. From (17) and (18) for equilibrium

$$p \leq x'\gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \dots \dots \quad (22)$$

and

$$p' \geq x''\gamma \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^2 \dots \dots \quad (23)$$

Combining (22) and (23) in the value of p ,

$$p = \frac{p + p'}{2} = \frac{\gamma}{2} \left\{ x' \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)^2 + x'' \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 \right\}. \quad (24)$$

Having assumed the values of γ and ϕ for any particular case, the above formulas determine the permissible magni-

tudes of the intensities at the heel and toe of the foundation for any depth. The breadth of the base of the foundation may now be assumed, and the actual intensities compared with those permissible; if p is too large or p' too small, another trial must be made. Usually one or two trials are sufficient. If one prefers to compute the width of the base of a trapezoidal foundation, the formula given below can be employed.

Determination of the breadth B'' of a trapezoidal foundation for a given loading and a maximum intensity p at the toe. (Back of foundation vertical.)

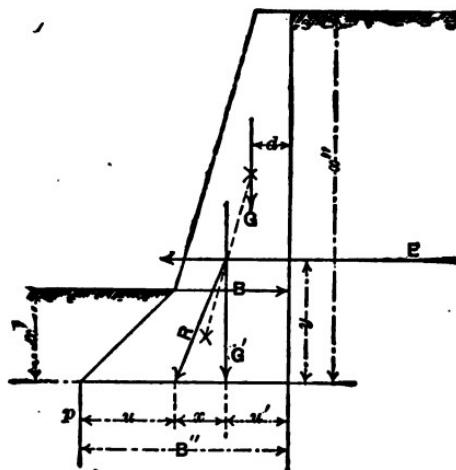


FIG. 18.

Let G = total vertical weight supported by top of foundation;

E = thrust of earth;

p = maximum intensity of pressure at toe of foundation as found from (22);

and B'' = breadth of base of foundation.

Then

$$B'' = \frac{-G + \sqrt{p(6dG + x'B^*W + 6Ey) + G^2}}{p}. \quad (25)$$

The foundation can nearly always be designed as a trapezoid having a vertical back, and then if necessary the batter in front can be stepped. For walls under twenty feet in height, retaining material which will assume a slope of $1\frac{1}{2}$ to 1, the most economical foundation is rectangular in section if the base must be four feet deep to escape the action of frost. Where frost need not be considered, of course more shallow and broader foundations can be employed.

Abutting Power of Earth.—Let the surface of the earth be horizontal and the body pushing the earth have a vertical face; then at the depth x' the maximum horizontal pressure per unit of area is (see Case I above)

$$q = x'\gamma \frac{1 + \sin \phi}{1 - \sin \phi}$$

and since q varies directly as x' , the total thrust P which the earth is capable of resisting is

$$P = \frac{(x')^2 \gamma}{2} \frac{1 + \sin \phi}{1 - \sin \phi} \quad \dots \quad (26)$$

Bearing Power of Earth.—The bearing power or the intensity of the pressure which earth can resist depends not only upon the character of the earth, but upon the depth to which the foundation is extended, as shown by the formulas for p given above. For example, the foundation may be very broad and shallow or quite narrow and deep. The

intensity of the pressure in the first case being considerably smaller than in the second, and both conditions fulfilling the conditions of stability. It appears then that the bearing powers of earth given by various writers must be employed with caution, unless the conditions upon which the values were based are known.

APPLICATIONS.

The determination of the earth-pressure by the preceding formulas and graphical constructions is a very simple operation when the angle ϕ has been determined or assumed. That care and judgment be used in assuming the value of ϕ is very important, since a change of a few degrees in the value of ϕ sometimes causes a large change in the value of E . An inspection of Diagram I shows that the value of the coefficient A increases very rapidly as ϕ decreases.

When the earth to be retained contains springs, the bank must be thoroughly drained if it is to be retained by an economical tight wall; if it is not drained, the angle ϕ will be likely to become very small as the earth becomes wet.

When the location of the earth to be retained is subjected to jars, the value of ϕ will be decreased.

Hence, in assuming the value of ϕ , the engineer must be sure that the value assumed will be the least value which, in his judgment, it is likely to have.

In constructing the wall the judgment and authority of the engineer must again be exercised in order that the wall be constructed as designed.

In all cases, to insure perfect drainage between the back

of the wall and the earth, numerous "weep-holes" should be provided in the body of the wall, or proper arrangements made to carry away the water at the base of the wall. To facilitate drainage, the backing resting against the wall should be sand or gravel.

In no case should water be permitted to get under the foundation of the wall, neither should the earth in front of the wall be allowed to become wet.

In cold localities the back of the wall near the top should have a large batter to prevent the frost from moving the top courses of stone. As a guard against sliding, the courses of the wall should have very rough beds. The strength of a wall is increased the nearer it approaches a monolith.

Care should be taken to have the foundation broad and deep enough to prevent sliding and upheaving of the earth in front. In clay the foundation should be deep, while in sand or gravel it may be broad and shallow.

The following examples illustrate the application of the formulas:

Ex. 1. Design a trapezoidal wall of sandstone, weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining forward 5° , to retain a bank of sand sloping upward at an angle of 20° .

Data.

$$\gamma = 100 \text{ lbs.}, W = 150 \text{ lbs.}; \epsilon = 20^\circ, \phi = 39^\circ, \alpha = 5^\circ; H = 30 \text{ ft.}, B' = 3 \text{ ft.}, x = 2.63 \text{ ft.}$$

1°. Graphical determination of the values of E and δ .

The graphical solution of the problem is shown in Fig. 19, where E is found to equal 15,000 pounds. δ lies between 35° and 36° .

2°. Algebraic determination of E and δ .

$$E = \frac{H^2 \gamma}{2} (B) \sqrt{(C) + (D)A^2 + (E)A} \dots \dots \dots (1')$$

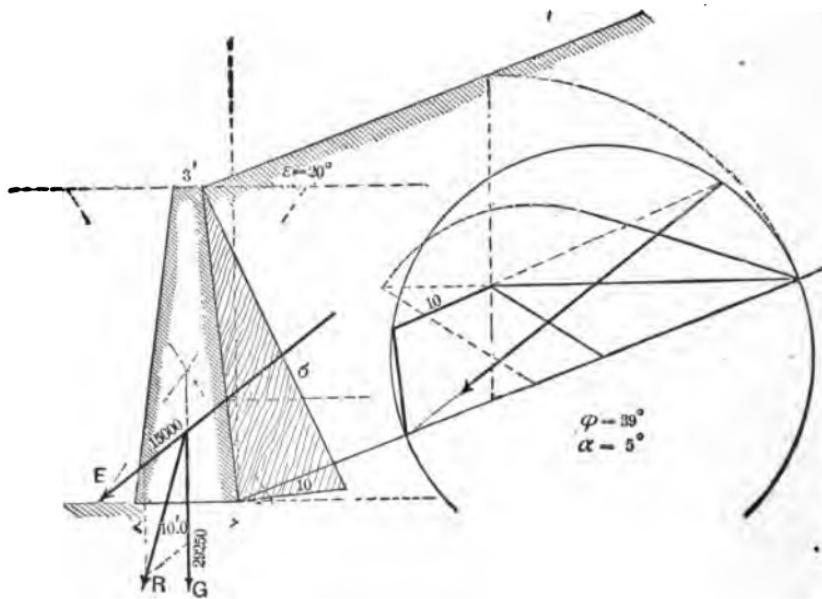


FIG. 22

Substituting the values of B , C , D , and E as given in the tables, and that of A as given by Diagram I, this becomes

$$E = \frac{900 \times 100}{2} (1.036) \times \sqrt{0.008 + (1.057)(0.264)^2 + (0.061)0.264},$$

$$E = 45,000 (1.036) \sqrt{0.098} = 14,500 \text{ lbs.}$$

$$\tan \delta = \frac{\sin \alpha}{\cos (\epsilon - \alpha) A} + \tan \epsilon, \dots \quad (2'a)$$

$$\tan \delta = \frac{0.087}{0.966(0.264)} \doteq 0.364,$$

$$\tan \delta = 0.705 = \tan 35^\circ 11', \text{ about.}$$

3°. Algebraic determination of the value of B under the assumption that $Q = \frac{1}{2}B$.

$$\begin{aligned} B'' + B \left\{ \frac{4E}{HW} \sin \delta + B' - x \right\} \\ = \frac{2E}{HW} \left\{ H \cos \delta + x \sin \delta \right\} + 2B'x + B'' . \quad (8) \end{aligned}$$

$$\begin{aligned} B'' + B \left\{ \frac{4 \times 14500}{30 \times 150} 0.576 + 3 - 2.63 \right\} \\ = \frac{2 \times 14500}{30 \times 150} \{ 30 \times 0.817 + 2.63 \times 0.576 \} + 6 \times 2.63 + 9, \end{aligned}$$

$$B'' + 7.79B = 172.53,$$

$$B = -3.89 \pm \sqrt{172.53 + 3.9^2};$$

$$\therefore B = 13.69 - 3.89 = 9.80 \text{ ft.};$$

or, practically, 10 feet is the required width of the base.

4°. To determine if the wall will slide on a foundation of sandstone.

From (14),

$$\tan \phi' \geq \frac{E \cos \delta}{G + E \sin \delta}.$$

$$\text{Taking } B = 10 \text{ ft.}, \quad G = \frac{10 + 3}{2} 30 \times 150 = 29250 \text{ lbs.}$$

$\delta = 35^\circ 11'$, $\cos \delta = 0.817$, and $\sin \delta = 0.576$, then

$$\frac{E \cos \delta}{G + E \sin \delta} = \frac{14500 \times 0.817}{29250 + 14500 \times 0.576} = 0.315.$$

From Table II, the value of $\tan \phi'$ for masonry is 0.6 to 0.7; hence there is no danger of the wall sliding on the foundation.

According to the *Engineering News* formula the base of this wall would be $\frac{1}{3}H$ "plus a few inches for good luck," or about 13 feet, and by the old rule of one third the height 10 feet.

Ex. 2. Design a trapezoidal wall of sandstone weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining backward 15° , to retain a bank of sand sloping upward at an angle of 30° .

Data.

$\gamma = 100$ lbs., $W = 150$ lbs.; $e = 30^\circ$, $\phi = 33^\circ$, $\alpha = -15^\circ$; $H = 30$ ft., $B' = 3$ ft., $x = 8$ ft.

1°. Graphical determination of the values of E and δ .

In Fig. 20, let BG represent the surface of the earth, and AB the back of the wall. Draw AF parallel to BG , and from any point D' in AF lay off $D'F$ equal to the vertical $D'G$, and draw FL horizontal; lay off the angle $IFD' = \phi = 33^\circ$, and locate the point M in $D'F$ so that if an arc be described with M as a centre and LM as a radius the arc will be tangent to IF ; then with M as a centre and MF as a radius, describe the circumference FHJ and draw JH

parallel to AB ; at A draw AL perpendicular to AB and equal to HI . Then

$$\frac{(AB)(AL)}{2} \gamma = \frac{(30.9)(9.6)}{2} 100 = 14800 = E.$$

To determine δ , prolong HI to K and draw KJ . Then the angle which this line makes with the horizontal is equal to δ , which is 6° to 7° in this case.

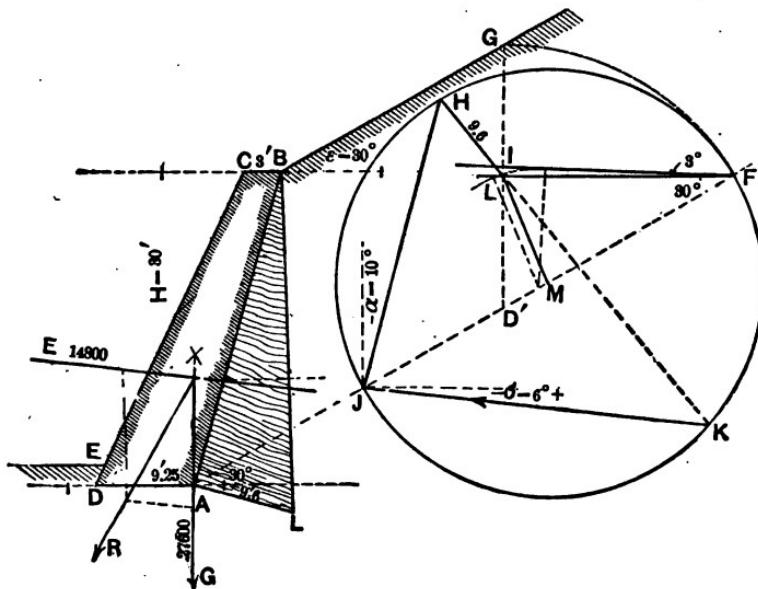


FIG. 20.

2°. Algebraic determination of E and δ .

Substituting in (1) and remembering that α is negative,

$$E = 45000 (0.875) \sqrt{0.067 + 0.183 - 0.111} = 14600 \text{ lbs.}$$

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From (1'a),

$$\tan \delta = \frac{-0.259}{0.707(0.524)} + .577 = -0.123 = \tan (-7^\circ).$$

3°. *Algebraic determination of the value of B under the assumption that Q = $\frac{1}{3}B$.*

Substituting the proper values in (8) and remembering that α is negative,

$$B = -4.7 \pm \sqrt{163.44 + (4.7)^2} = 9.0 \text{ ft.}$$

Ex. 3. Determine the dimensions of a brick wall having a vertical back to retain a bank of sand sloping upward at an angle of 20° . $\phi = 30^\circ$, $H = 20'$, $B' = 2'$, $\gamma = 100$.

1°. *Algebraic determination of E and δ.*

Since $\alpha = 0$,

$$E = \frac{H \gamma}{2} A \dots \dots \dots \quad (2)$$

$$E = \frac{400 \times 100}{2} 0.424 = 8480; \text{ say, } 8500 \text{ lbs.}$$

The value of A is readily found from Diagram I.

$$\delta = \epsilon = 20^\circ, \text{ since } \alpha = 0.$$

2. *Algebraic determination of the value of B under the condition that Q = $\frac{1}{3}B$.*

$$B^2 + B \left\{ \frac{4E}{HW} \sin \delta + B' \right\} = \frac{2E}{W} \cos \delta + B'^2. \quad (9)$$

From Table I, $W = 125$ lbs. Then

$$B^* + B \left\{ \frac{4 \times 8500}{20 \times 125} 0.342 + 2 \right\} = \frac{2 \times 8500}{125} 0.940 + 4,$$

or

$$B^* + 6.65B = 131.84.$$

$$B = -3.33 \pm \sqrt{131.84 + 3.33^2},$$

and

$$B = -3.33 + 11.94 = 8.61 \text{ ft.}$$

Ex. 4. Determine the value of B in Ex. 3 under the assumption that $\epsilon = 0$ (horizontal earth-surface).

$$E = \frac{H^* \gamma}{2} \left\{ \tan^2 \left(45^\circ - \frac{\phi}{2} \right) = \frac{1 - \sin \phi}{1 + \sin \phi} \right\}, \quad \dots \quad (6)$$

or $E = 20000 (0.333) = 6666$, say 6700 lbs.

Since $\alpha = 0$, and $\epsilon = 0$, $\delta = 0$,

$$B^* + BB' = \frac{2E}{W} + B'^*; \quad \dots \quad \dots \quad \dots \quad (10)$$

$$B^* + 2B = 111.2;$$

$$B = -1 \pm \sqrt{111.2 + 1},$$

and

$$B = -1 + 10.59 = 9.6 \text{ ft.}$$

Ex. 5. Determine the value of B in Ex. 3, under the assumption that $\epsilon = \phi = 30^\circ$.

$$E = \frac{H^* \gamma}{2} \cos \phi = 20000 (0.866) = 17320 \text{ lbs.}$$

From (9),

$$B^* + B \left\{ \frac{4 \times 17320}{20 \times 125} 0.5 + 2 \right\} = \frac{2 \times 17320}{125} 0.866 + 4;$$

$$B^2 + 15.86B = 244.05;$$

$$B = -7.93 + \sqrt{244.05 + 7.93^2}.$$

and $B = -7.93 + 17.52 = 9.6$ ft.

Ex. 6. Determine the resultant pressure against the back of a wall when the surface of the earth carries a load equivalent to 5 feet in depth of sand.

$H = 30$ ft., $\alpha = 10^\circ$, $\phi = 30^\circ$, $\epsilon = 0$, and $\gamma = 100$ lbs.

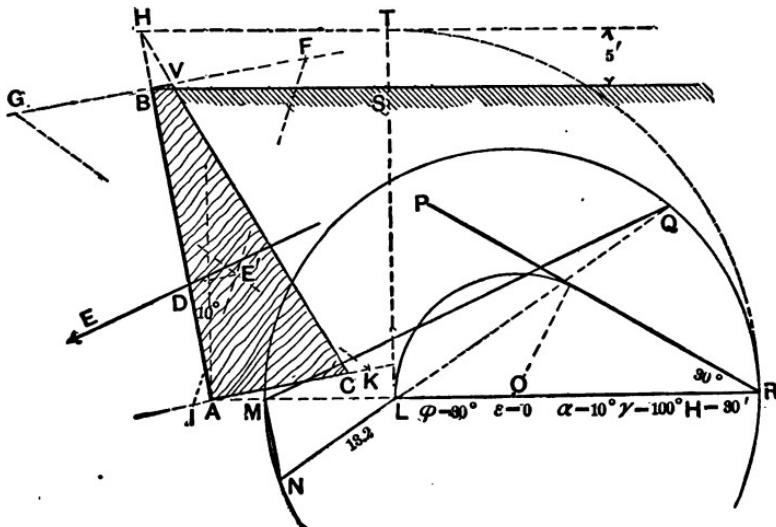


FIG. 21.

Graphical solution of the problem.—In Fig. 21, let BS represent the surface of the earth, and BA the back of the wall.

Make $ST = 5$, and draw HT and BH . Draw AR parallel to BS , parallel to HT , and make LR equal to LT ; lay off the angle LRP equal to 30° ; with O as a centre

draw an arc passing through L tangent to PR , and then with OR as a radius describe the circumference of the circle RQM , and at M draw MN parallel to AH ; at A and normal to AH draw AC equal to NL . Then

$$\frac{AC + BV}{2} BA \cdot \gamma = E.$$

The direction of E will be parallel to QM .

To determine the point of application of E , find the centre of gravity E' of $ABVC$, and draw $E'D$ parallel to AC , then D will be the point of application of E .

E' can be found as follows: Produce AC and BV , make $AI = CK = BV$, $BG = VF = AC$, and join F and I and G and K . Then E' , the intersection of FI and GK , will be the centre of gravity of $ABVC$. BD can be found from the formula

$$BD \cos 10^\circ = \frac{2(TL)^2 - (TL)(TS) - (TS)^2}{3(TS + TL)}.$$

Ex. 7. Determine graphically the value of E when $\epsilon = 0$ and $\alpha = 0$, ϕ , γ , and H being given.

In Fig. 22 let BF represent the surface of the earth, and AB the back of the wall. Draw AL parallel to BF and make $IL = IF$; lay off the angle $GLH = \phi$, and at any point K in LH draw MK perpendicular to HL , and lay off $MO = MK$; draw MJ parallel to OI . Then will the arc IN , described with J as a centre and IJ as a radius, pass through I and be tangent to GL ; with J as a centre and JL as radius describe the circumference LH ; at A lay off $AC = HI$ and normal to AB . Then

$$\frac{AC \times AB}{2} \gamma = E.$$

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E is parallel to BF and applied at D , AD being equal to $\frac{1}{2}AB$.

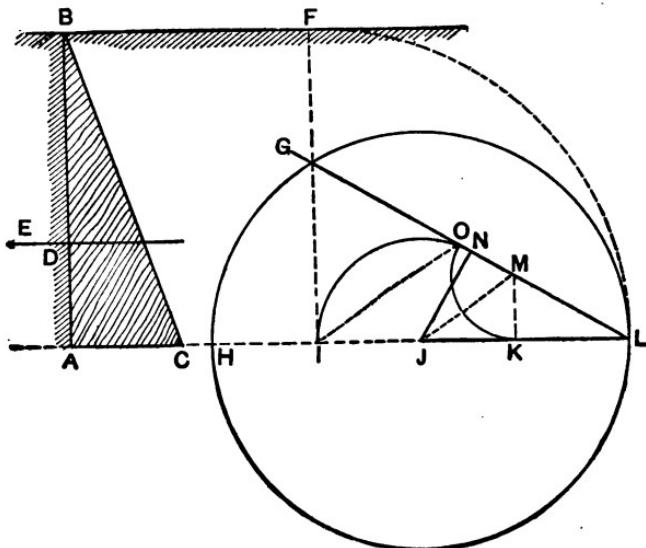


FIG. 23.

Ex. 8. Determine the earth-thrust on the profile shown in Fig. 23, H , γ , ϕ , and ϵ being given.

Graphical solution of the problem.—Let $BCDEA$ represent the given profile, and let the surface of the earth be horizontal. Prolong BC until it intersects SA in S ; draw SR normal to BCS and equal to the intensity of the earth-pressure at S ; connect B and R . Then from the middle point of BC draw GF parallel to SR ; the distance GF multiplied by γ will be the average intensity of the earth-pressure on BC . In a similar manner the average intensities on CD , DE , and EA can be found, and hence the total pressures on each determined. The points of application of these resultant pressures, E_1 , E_2 , E_3 , and E_4 ,

can be found by the method used in Ex. 6 for finding the centre of gravity of a trapezoid. The directions of

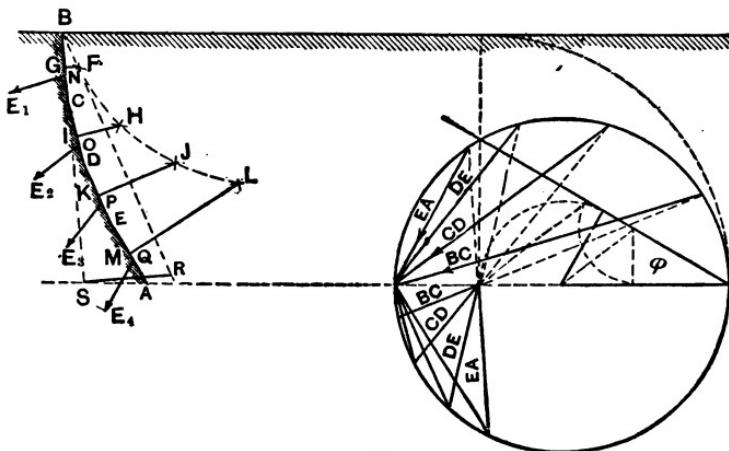


FIG. 28.

E_1 , E_2 , E_3 , and E_4 are found from the construction on the right.

Ex. 9. Determine the thrust of the earth against a vertical wall when ϵ is negative.

For the explanation of this construction, see page 21, Fig. 9.

Ex. 10. From the following data determine E , δ , and Q :

$$\epsilon = 0, \phi = 38^\circ, \alpha = 10^\circ 23'; \gamma = 90 \text{ lbs.}, W = 170 \text{ lbs.};$$

$$H = 15 \text{ ft.}, B = 6 \text{ ft.}, B' = 2 \text{ ft.}$$

$$Ans. E = 3037 \text{ lbs.}, \delta = 37^\circ 37', Q = 2.2 \text{ ft.}$$

Ex. 11. Determine the dimensions of a trapezoidal wall built of dry, rough granite, having a vertical back and being 20 feet high, to safely retain the side of a sand cut,

the surface of the sand being level with the top of the wall. $W = 165$ lbs., $\gamma = 100$ lbs., $\phi = 33^\circ 40'$, $H = 20$ ft., $B' = 2$ ft.

Ans. $E = 5734$ lbs., $\delta = 0$, $B = 8$ ft., and $Q = 2.8$ ft., about.

Ex. 12. The same as Ex. 11, with $\alpha = 8^\circ$ instead of $\alpha = 0$.

Ans. $E = 6330$ lbs., $B = 9.98$ ft., and $Q = 2.7$ ft.

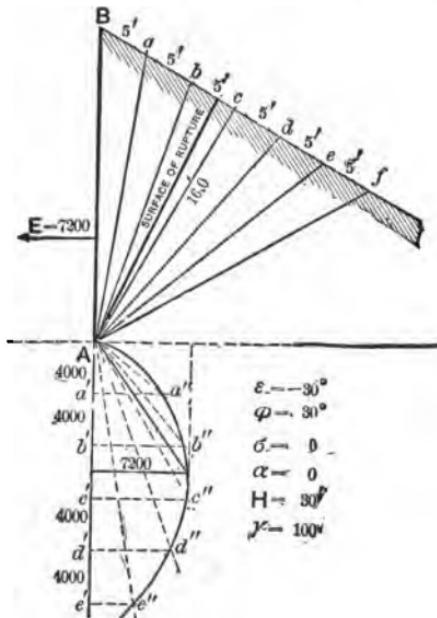


FIG. 24.

Ex. 13. What must be the dimensions of a rubble wall of large blocks of limestone, laid dry, to retain a sand filling which supports two lines of standard-gauge railroad track? (Assume the depth of sand to produce a pressure on the earth equal to that produced by the railroad and trains as 4 feet.)

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$H = 15$ ft., $\alpha = 8^\circ$, $\phi = 33^\circ 40'$, $\gamma = 100$ lbs., $W = 170$ lbs., $B' = 3.5$ ft.

Ans. $E = 5760$ lbs., $\delta = 26^\circ 7'$, $B = 8$ ft., $Q = 2.7$ ft.

Ex. 14. Determine E , δ , B , and Q , when $W = 170$ lbs., $\gamma = 100$ lbs., $\alpha = 8^\circ$, $\epsilon = \phi = 33^\circ 40'$, $H = 20$ ft., $B' = 2$ ft.

Ans. $E = 21760$ lbs., $\delta = 32^\circ 25'$, $B = 9$ ft., $Q = 3$ ft.

* Ex. 15. A wall 9 ft. high faces the steepest declivity of earth at a slope of 20° to the horizon; weight of earth 130 lbs. per cubic foot, angle of repose 30° . Determine E when $\alpha = 0$.

Ans. $E = 2187$ lbs.

* Ex. 16. $\epsilon = 33^\circ 42'$, $\phi = 36^\circ$, $H = 3$ ft., $\gamma = 120$ lbs., $\alpha = 0$. Determine E .

Ans. $E = 278$ lbs.

* Ex. 17. $\phi = 25^\circ$, $\epsilon = 0$, $\alpha = 0$, $H = 4$ ft., $\gamma = 120$ lbs., $E = ?$

Ans. $E = 390$ lbs.

* Ex. 18. $\phi = 38^\circ$, $\epsilon = 0$, $\alpha = 0$, $H = 3$ ft., $\gamma = 94$ lbs., $E = ?$

Ans. $E = 100.5$ lbs.

* Ex. 19. A ditch 6 feet deep is cut with vertical faces in clay. These are shored up with boards, a strut being put across from board to board 2 feet from bottom, at intervals of 5 feet apart. The coefficient of friction of the moist clay is 0.287, and its weight 120 lbs. per cubic foot. Find the thrust on a strut, also find the greatest thrust which might be put upon the struts before the adjoining earth would heave up.

Ans. $E = 1230$ lbs.

Thrust per strut = 6128 lbs.

Greatest thrust = 19029 lbs.

58 FOUNDATIONS FOR WALLS RETAINING EARTH.

Ex. 20. Examine the stability of the wall shown in Fig. 25, and design a foundation which will be safe as long as the condition of the earth remains unchanged; the weight of the masonry being 145 pounds per cubic foot, that of earth 100 pounds, and the angle of repose of the earth such that it will stand at a slope of $1\frac{1}{2}$ to 1.

Stability of the Wall upon the Foundation.—Replacing the stepped back by the line BD , the thrust of the earth is found to be about 9900 pounds. The direction of this force is shown in Fig. 25; since it cuts the base of the wall there is no danger of the structure being overturned, however large E may become.

Determining the centre of gravity of the wall and also

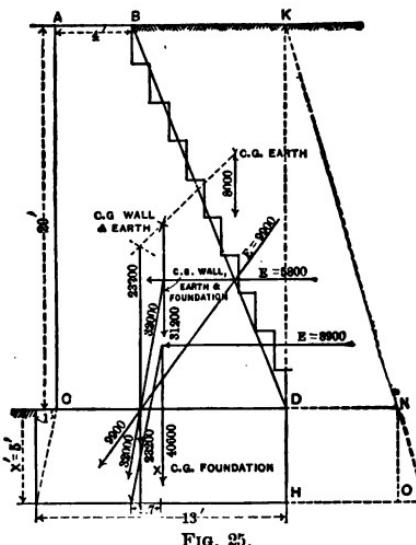


FIG. 25.

its weight, and combining this with E , the resultant pressure upon the base of the wall is found to be about 32,000 pounds. This resultant makes an angle of less than 11 degrees with the normal to the base. Now since for masonry sliding upon masonry the angle of friction is from 31 to 35 degrees (Table II), there is no danger of failure by sliding upon the foundation.

Failure by the crushing of the material need not be con-

cerned in this case, as the entire base is subjected to compression.

sidered, as the maximum intensity of the pressure upon the base is many times smaller than the ultimate strength of the material. See page 68.

The resultant pressure upon the base can be found also by assuming the earth on the left of the vertical to be supported by the wall, and that the pressure of the earth upon the right of this line acts against the vertical plane KD ; this pressure is about 5800 pounds, and is horizontal. Combining this force with the weight of the wall and earth on the left of the line KD , the resultant pressure upon the base is found to be the *same in magnitude and direction as by the first method.*

The Foundation.—The depth of the foundation must be below the action of frost; let this be assumed as 5 feet; then by (22), with $x' = 5$ feet, the *maximum* allowable pressure at the toe of the foundation is about 6000 pounds per square foot, and by (23) the *minimum* allowable pressure is about 200 pounds for $x'' = 25$ feet.

Assuming that the foundation is vertical at the back and trapezoidal in section, the length of the base B'' can be found from (25), which will satisfy the condition of maximum pressure at the toe. Letting $p = 5000$ and $x' = 5$, and solving (25), B'' is found to be between 12 and 13 feet; say 13 feet.

To determine if this width is sufficient to satisfy all the conditions of equilibrium, the resultant of all forces acting upon the base must be found.

* The total earth-pressure upon the vertical HK is about 8900 pounds. Combining this with the weight of the wall, earth supported by the wall, and that of the foundation, the resultant vertical pressure is found to be about 40,600

* The pressure again t the foundation in front of the wall has been neglected, but can be easily included by taking the area $KHON$ instead of KHP .

pounds, and is applied within the middle third of the base, about 1.7 feet to the left of the centre.

The intensity of the pressure at the toe is (page 31)

$$p = \left\{ 1 + \frac{6(1.7)}{13} \right\} \frac{40600}{13} = \text{about } 5600 \text{ pounds,}$$

which is less than the maximum allowable intensity. The intensity at the heel is $p' = 2p$, $\therefore p = \text{about } 600 \text{ pounds}$, which is greater than the minimum allowable intensity; hence this foundation is sufficient to prevent settlement.

A glance at Fig. 25 is sufficient to show that the foundation will not slide upon the earth even if the movement were not opposed by a force of some 4000 pounds, being the abutting power of the earth in front of the foundation.

The above foundation then fulfils all the conditions of stability, but to allow for contingencies the foundation should be designed under the assumption that ϕ may be somewhat smaller than its average value, which is equivalent to broadening the base if the depth remains the same.

NOTE.—Although the above discussion considers a foundation as safe if the pressures at the heel and toe are within the theoretical requirements, yet, owing to the compressibility of most soils met with in practice, it is advisable to so design the foundation that the resultant pressure cuts its base as near the centre as possible. This will give more nearly a uniform pressure on the soil and, consequently, there will be less probability of the wall and foundation as a whole tipping forward.

It is seldom the case that walls fail in themselves, but there are almost innumerable cases where walls lean forward due to poor foundations and foundations so constructed that there is a wide variation of pressure upon the soil from heel to toe.

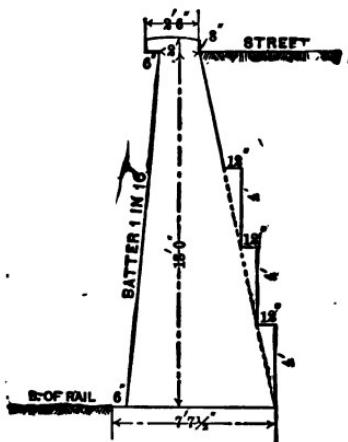
EXAMPLES OF RETAINING-WALL PROFILES.

FIG. 26.

A Standard Profile used for the past twenty years near New York City, where railway tracks have been lowered below the streets.
(*Engineering News*, 1889.)

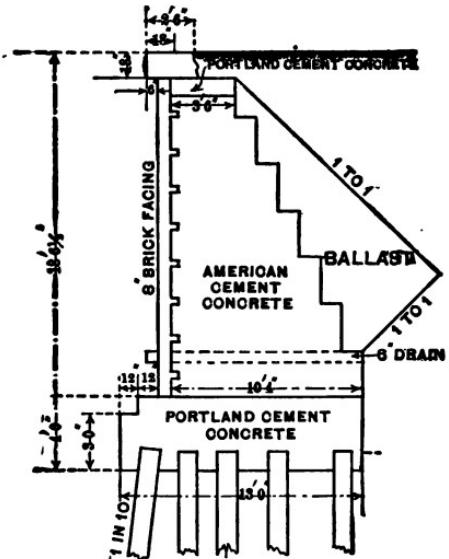


FIG. 27.

Profile of Retaining-wall at Ferdinand Street Bridge, Boston, Mass.
(*City Engineer's Report*, 1891.)

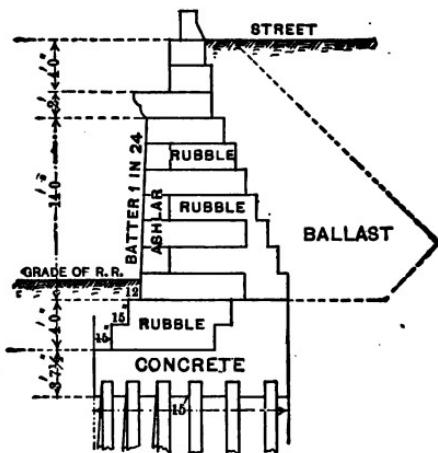


FIG. 28.

Profile of Abutment at Ferdinand Street Bridge, Boston, Mass.
(*City Engineer's Report, 1891.*)

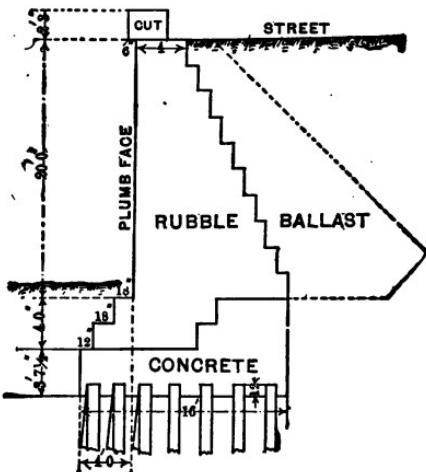


FIG. 29.

Profile of Retaining-wall at Boylston Street Bridge, Boston, Mass.
(*City Engineer's Report, 1888.*)

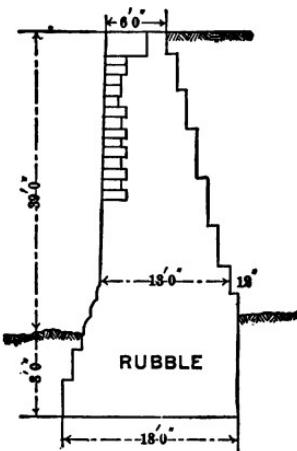


FIG. 30.
Profile of Retaining-wall at Liverpool, England. (*Harcourt*).

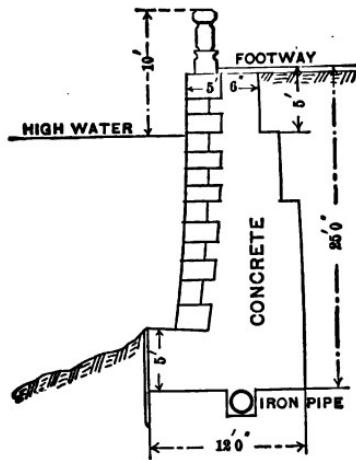


FIG. 31.
Profile of Retaining-wall, Thames Embankment, Chelsea. (*Harcourt*.)

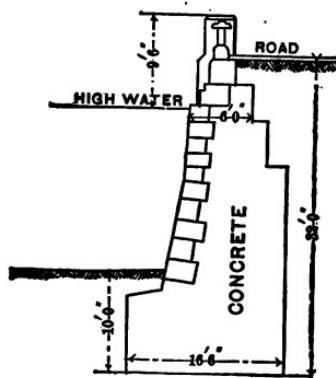


FIG. 32.

Profile of Retaining-wall Thames Embankment, Lambeth. (*Harcourt.*)

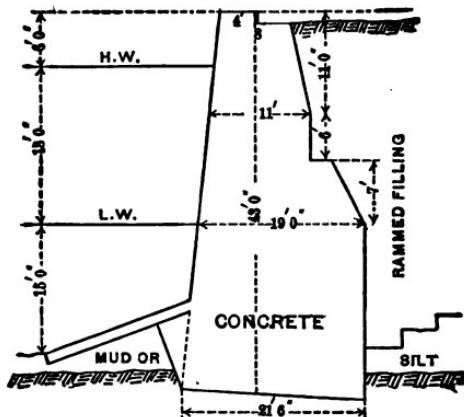


FIG. 33.

Profile of Concrete Retaining-wall at Chatham. (*Harcourt.*)

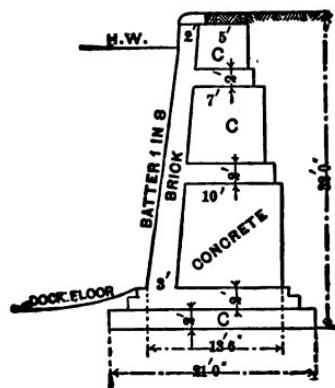


FIG. 34.
Profile of Retaining-wall at Millwall. (*Harcourt*)

FOUNDATIONS.

The proper proportions of foundations to suit different conditions have been the results of experience principally, though theory enters into their design in many ways. Under certain logical assumptions, the offsets of wood, iron, or stone foundation courses can be as accurately determined as the stresses in any beam subjected to cross-bending. The strengths of various materials which enter into the construction of foundations have been fairly well determined experimentally, so that the allowable intensities of the pressures, and consequently the areas of the foundation courses, can be accurately determined. There remains the most difficult portion to be decided, namely, the proper intensity of the pressure upon the earth which must support the load. Under certain assumptions this can be computed, but the best of judgment must be exercised in making the assumptions upon which calculations are based.

Whenever possible, the intensity of the pressure upon the earth should be uniform under all parts of the structure (assuming the earth to be homogeneous), and the foundations extend to the same depth. Theoretically, a greater intensity is allowable at a greater depth, but practically this may lead to unequal settlement, due to the compressibility of the earth, which theory does not take into account.

FOUNDATIONS UPON ROCK.

In preparing a bed for the structure to be erected all loose and decayed parts of the rock must be removed, and the surface made as nearly horizontal as practicable; when the surface is inclined, it may be cut into steps with horizontal

and vertical faces; if holes exist, they may be filled with concrete. In some cases a proper surface for supporting the proposed structure can be secured by covering the rock surface with a layer of concrete, which may vary from a few inches to two or more feet in thickness. (Figs. 39 and 42.)

The *maximum* intensity of the pressure upon a rock foundation should not exceed *one sixth* the crushing strength of the rock for a steady and uniform load, or one tenth the crushing strength for a load due to the weight of the structure plus a varying load such as is caused by wind or earth pressure.

In no case should any portion of the horizontal joints be subjected to tension. The maximum deviation of the centre of pressure from the centre of gravity of the base section, when the section is a symmetrical figure, can be found from the formula

$$x_0 = \frac{I}{Ay}, \text{ (Rankine);}$$

where x_0 = the maximum deviation sought;

I = the moment of inertia of the section relative to an axis perpendicular to the direction in which the maximum deviation is sought;

and y = the distance from the centre of gravity of the section to the edge furthest from the centre of pressure measured along an axis passing through the centre of pressure and the centre of gravity.

Following are the more common sections of foundations with the corresponding values of x_0 :

Rectangle... $A = bh$, $x_0 = \frac{1}{6}b$;

Circle..... $A = \pi r^2$, $x_0 = \frac{1}{8}d$;

Hollow rectangle:

$$A = bh - b'h', \quad x_0 = \frac{b}{6} \left(1 - \frac{b'h''}{bh} \right) \div \left(1 - \frac{b'h'}{bh} \right);$$

Hol. square. $A = h^2 - h'^2, \quad x_0 = \frac{h}{6} \left(1 + \frac{h''^2}{h^2} \right);$

Hol. circle. $A = \pi(r^2 - r''^2), \quad x_0 = \frac{d}{8} \left(1 + \frac{r''^2}{r^2} \right).$

The ultimate compressive strengths, in pounds per square inch, of various rocks used in foundations are, approximately, for

| | |
|--|----------------|
| Granite..... | 13000 to 26000 |
| Sandstone | 7000 " 14000 |
| Soft sandstone..... | 3000 |
| Strong limestone..... | 6000 " 23000 |
| Weak limestone..... | 3000 |
| Hard red brick..... | 6000 " 10000 |
| Paving brick..... | 6000 " 13000 |
| Portland cement concrete, 1:3:6, one month old... | 2000 |

See Table I for additional information.

FOUNDATIONS UPON EARTH.

Firm Earth.—Earth which has an angle of repose of at least 27° may be considered as firm, and for foundation purposes requires little preparation other than the excavation of a trench or pit, and making the surface receiving the masonry level. From Table II it is seen that sand, gravel, and damp clay are classed as firm soils; however,

these may become so saturated with water that their angles of repose will become considerably less than 27° , hence precautions must be taken against too much water by draining the ground in the immediate vicinity of the foundation. Particular care must be taken in the case of clay, or sand which will become a kind of quicksand when saturated with water.

Before attempting to design a foundation, the character of the earth must be determined either by test excavations, borings, or from the experience of others. It often happens that from all surface indications the earth appears to be firm, but upon excavating it is found there is a stratum of semi-fluid mud or quicksand underneath; in such cases care must be taken to determine the minimum thickness of the stratum of firm earth, for if too thin it will not be safe to build upon, and then a foundation has to be prepared according to some of the methods described later.

Considering the earth as a homogeneous granular mass, the supporting power at any depth can be computed when the angle of repose ϕ is known. Some practical men object to any theoretical formulas being employed in connection with the determination of the bearing or supporting power of earth, claiming that the assumptions upon which the formulas are based are rarely if ever found in practice. This is probably true to a certain extent, yet the theoretical formulas are upon the safe side, and do not lead to absurd results; in fact, the results obtained by their judicious application agree very well with the practice of the best engineers.

If p = the maximum supporting power per square foot of earth;

γ = the weight of one cubic foot of earth;

ϕ = the angle of repose;

and x' = the depth of the plane below the surface upon which the maximum supporting power is desired;

then

$$p = x' \gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^* \text{ (see page 40). . . (1)}$$

And if p' is the minimum intensity of the pressure upon the earth which is allowable for the stability of the earth and the foundation with its load,

$$p' = x'' \gamma \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^* \text{ (see page 40), . . (2)}$$

where x'' is the depth of the plane considered below the surface of the earth.

The above equations neglect any friction between the earth and the masonry of the foundation. In deep foundations this is a large factor on the safe side.

If the surface of the earth is level, then $x' = x''$; and further, if the earth is subjected to a uniformly distributed load only the average intensity need be considered.

Equation (2) is considerably different from that given by Rankine, and writers who have followed him, in this, that they consider the minimum intensity allowable to be equal to $x'' \gamma$ = the average intensity of the pressure upon a plane at a depth x'' in an unlimited mass. This does not appear to the writer to be a logical treatment of the subject, if the mass has an angle of repose greater than zero, and the maximum intensity allowable be determined as a function of this angle.

According to the assumption of Rankine, it would appear that if a box without a bottom were sunk into a mass of perfectly dry sand it would be filled from the bottom until

the surfaces without and within were at the same level; but this does not take place, and would not even if the sides of the box were frictionless. The sand only fills the box partially, or until the requirements of equation (2) are fulfilled. Hence it seems to the writer that if the maximum intensity is a function of ϕ , the value of the minimum intensity must be also.

From equations (1) and (2) it is evident that the allowable intensity upon the earth of any pressure or load commonly called the supporting power varies *directly as the depth*, as long as ϕ remains unchanged; hence all tables of supporting powers of earth are of little value unless the depth of the foundation upon which they are based is known. Unfortunately this is omitted in most cases, and only the character of the earth is given. The depth to which foundations must be sunk in many localities has a *minimum* value governed by the depth to which frost extends. This is not always true, however, as in Terre Haute, Indiana, frame houses and brick blocks two and one-half stories high are constructed practically upon the surface, the sod only being removed. The width of the foundation is not excessive, but on the contrary narrow. No serious settlement results, owing to the character of the earth, which is very sandy, and will not retain sufficient moisture to permit frost action to heave the structures. The actual load per square foot supported by the soil is about one ton. If x' be taken as one foot, γ as 100 pounds, and p as 2000 pounds, then from equation (1) ϕ is about 39° , which is below the actual value.

The above case, however, may be called an exception to the general rule that all foundations must be sunk below the action of frost, or to a depth of three feet or more according to the locality.

For convenience the values of

$$\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2 \text{ and } \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)^2$$

are given in the following table:

| ϕ | $\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2$ | $\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)^2$ | ϕ | $\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2$ | $\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)^2$ |
|--------|--|--|--------|--|--|
| 0 | 1.00 | 1.00 | 23 | 5.21 | 0.19 |
| 5 | 1.42 | 0.70 | 24 | 5.62 | 0.18 |
| 6 | 1.52 | 0.66 | 25 | 6.07 | 0.16 |
| 7 | 1.63 | 0.61 | 26 | 6.56 | 0.15 |
| 8 | 1.75 | 0.57 | 27 | 7.09 | 0.14 |
| 9 | 1.88 | 0.53 | 28 | 7.67 | 0.13 |
| 10 | 2.02 | 0.50 | 29 | 8.30 | 0.12 |
| 11 | 2.16 | 0.46 | 30 | 9.00 | 0.11 |
| 12 | 2.32 | 0.43 | 31 | 9.76 | 0.10 |
| 13 | 2.50 | 0.40 | 32 | 10.59 | 0.09 |
| 14 | 2.68 | 0.37 | 33 | 11.50 | 0.09 |
| 15 | 2.88 | 0.35 | 34 | 12.51 | 0.08 |
| 16 | 3.10 | 0.32 | 35 | 13.62 | 0.07 |
| 17 | 3.33 | 0.30 | 36 | 14.84 | 0.07 |
| 18 | 3.59 | 0.28 | 37 | 16.18 | 0.06 |
| 19 | 3.86 | 0.26 | 38 | 17.67 | 0.06 |
| 20 | 4.22 | 0.24 | 39 | 19.64 | 0.05 |
| 21 | 4.48 | 0.22 | 40 | 21.16 | 0.05 |
| 22 | 4.88 | 0.21 | | | |

Having determined upon the depth to which it is expedient to extend the foundation, a *minimum* value of ϕ must be assumed from a knowledge of the earth, and then the allowable bearing or supporting power can be found from equations (1) and (2); or if the supporting power is assumed, the minimum depth to which the foundation must be sunk can be found from the same equations.

The proper proportions of the foundation are most easily obtained from the following equations, which are deduced

for a few of the ordinary forms and conditions. All masonry foundations are usually trapezoidal in section, and hence formulas based upon this form can be applied to stepped foundations without serious error.

CASE I. *Given a uniformly distributed load to be supported by symmetrical trapezoidal foundation sunk to a known depth, to determine the minimum width of the base of the foundation.*

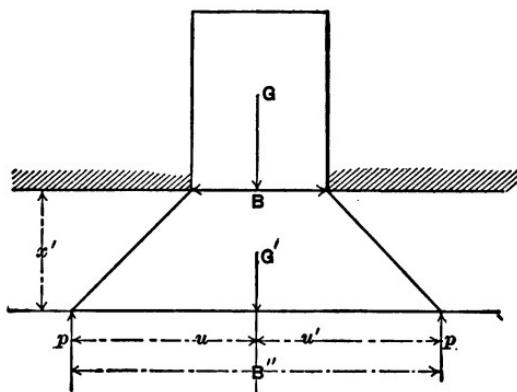


FIG. 85.
Section of Wall and Foundation.

Let G = the total weight to be supported less that of the foundation;

$G' = G + \text{weight of the foundation};$
and $B'' = \text{minimum breadth of the foundation}.$

Assuming x' , the value of p is

$$p = x' \gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^{\frac{1}{2}}.$$

From the figure

$$G' = G + W \frac{B + B''}{2} x' = B'' p;$$

or

$$B'' = \frac{2G + BWx'}{2p - Wx'}.$$

The above formula applies to a wall one foot long.—In case of an isolated pier, the value of x' can be found as above. B'' may be assumed and a rough calculation made to determine if the average pressure upon the earth is equal to or less than p . A second trial usually determines the proper value for B'' . The exact formula for the determination of the dimensions of a square or rectangular foundation with stepped sides is an equation of the second degree.

Ex. 1. A trapezoidal foundation 5 feet broad on top has to support 50,000 pounds per lineal foot in length, in earth having a minimum angle of repose of 30° . The maximum depth to which the foundation is to be sunk is 5 feet; determine B'' and p , when $\gamma = 100$ pounds and $W = 150$ pounds.

From (1)

$$p = 5 \cdot 100 \cdot 9 = 4500 \text{ pounds—say 4000;}$$

then

$$B'' = \frac{100000 + 3750}{8000 - 750} = 14.3;$$

or the proper width of the base is about 14.5 feet.

Ex. 2. A cast-iron plate, 2 feet square under a column, transmits a load of 20,000 pounds to a masonry foundation 3 feet square. How deep must this be sunk in earth when $\phi = 30^\circ$, $\gamma = 100$ pounds, and $W = 150$ pounds?

Neglecting the weight of the masonry in the foundation, the intensity of the pressure upon the earth is about 2200 pounds; then from (1) $x' =$ about 2.5 feet—say 3 feet.

The actual intensity of the pressure upon the earth is now $\frac{20,000 + 4050}{9} = 2670$ pounds. Substituting this value of p in (1) and solving for x' , its value is 2.96 feet; hence 3 feet is the required depth of the foundation.

The weight of the earth supported by the masonry of the foundation is neglected.

CASE II. *Unsymmetrical distribution of pressure upon the base of a foundation.*

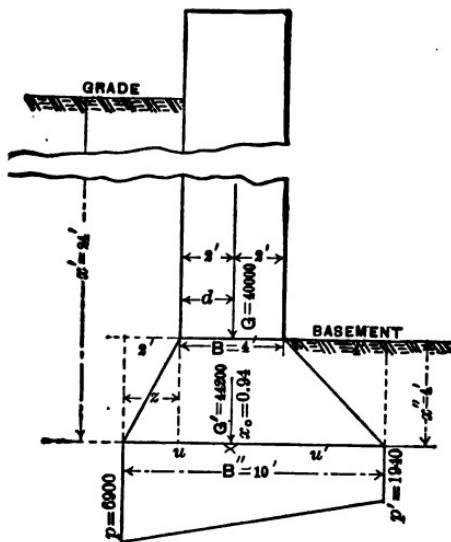


FIG. 33. Section of Wall and Foundation.

One of the many examples of pressure unevenly distributed upon the bed of a foundation is the case of an outside wall of a building located very near the property line and circumstances prevent encroaching upon the neigh-

boring property to any great extent. Here two conditions must be fulfilled. The maximum intensity of the pressure p , Fig. 36, must not be greater than the supporting power of the earth at the depth x' , and the minimum intensity p' must not be so small that the earth having a depth x'' may tend to heave the foundation.

Let p_0 = the *average* intensity of the pressure upon the base. Then

$$p_0 = \frac{G'}{B''} = \frac{p + p'}{2}, \quad p = x' \gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^{\frac{1}{2}}.$$

But

$$G' = G + \frac{B + B''}{2} x'' W.$$

Therefore

$$B'' = \frac{2G + B W x''}{2p_0 - W x''};$$

in which x'' is determined from the equation

$$p' = x'' \gamma \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^{\frac{1}{2}}.$$

It is thus possible to determine B'' quite easily, but the value of the offset z so that p and p' shall have their proper values must be either found by trial or computation. Since one or two trials are sufficient to determine z , the formula will not be given here.

Ex. 3. In Fig. 36, page 75, let $G = 40,000$ pounds, $B = 4$ feet, $d = 2$ feet, $x' = 24$ feet, and $x'' = 4$ feet. If the thrust of the earth be neglected, what must be the width of the base of the foundation, so that the average pressure per unit area shall not exceed 4800 pounds, and the maximum 7000 pounds, when $\gamma = 100$, $W = 150$,

$\phi = 30^\circ$? The bulk of the foundation to be on the right of the centre of the wall.

First determine the allowable intensities,

$$\text{max } p = x' \gamma(9) = 2400 \times 9 = 21600 \text{ pounds.}$$

$$\text{min} = x' \gamma(0.11) = 2400 \times 0.11 = 264 \quad "$$

$$\text{max } p' = x'' \gamma(9) = 400 \times 9 = 3600 \quad "$$

$$\text{min} = x'' \gamma(0.11) = 400 \times 0.11 = 44 \quad "$$

From the formula on page 76

$$B'' = \frac{2G + BWx''}{2p_0 - Wx''} = \frac{82400}{9000} = 9.15 \text{ feet.}$$

Take 10 feet as the value of B'' ; then the weight of the masonry in the foundation is 4200 pounds, and

$$p_0 = \frac{44200}{10} = 4420.$$

By graphics or by moments, assuming $z = 2$ feet, the resultant pressure cuts the base 0.94 foot from the centre, and hence $p = 6900$ pounds and $p' = 1940$ pounds.

The above width of base and the intensities just obtained satisfy all the conditions of the problem, though the value of z could be decreased a little, increasing the intensity at the toe and decreasing that at the heel.

Projection of Footing-courses.—Where masonry foundations are stepped as is the usual custom, the proper offset for each course may be determined as follows, by considering each offset as a cantilevered beam of stone *uniformly loaded*:

Let o = the offset of any particular course;

p_0 = the intensity of the pressure upon the base of the course;

t = the thickness of the course;
 R = the modulus of rupture of the material; and
 F = the factor of safety.

Then

$$p \cdot \frac{o^3}{2} = \frac{1}{6} \frac{R}{F} t^2,$$

or

$$o = t \sqrt{\frac{R}{F} \frac{1}{3p}}.$$

In case the intensity of the pressure is not uniform, but varies uniformly from one side to the other, the quantity p , may be replaced by p , the maximum intensity for the offset on the side having the greater pressure, and by p' , the minimum intensity for the steps or offsets on the side of the lesser pressure: in the first case the factor of safety will be slightly increased and in the second decreased.

The above formula is applicable only when the stones project less than half their length and when thoroughly well laid in cement mortar.

Other factors remaining the same, the offsets vary directly as the square roots of the moduli of rupture and inversely as the factors of safety, so that the table on page 79 can be applied for any values of R and F by simple proportion.

Foundations upon Soft Earth.—When a foundation must be placed upon soft earth which offers no particular difficulties other than the requirement of broadness or depth of the excavation, considerable expense can be avoided by excavating the soft material and replacing it by firm material, or by driving short piles spaced about

three feet on centres, commencing at the outer limits of the foundation and working towards the centre, and thus compressing the earth; sometimes holes are bored and filled with sand, making sand-piles, etc. The proper depth and spread of such foundations can be found from formulas (1) and (2) by including the prepared earth as a portion of the foundation.

**SAFE OFFSETS FOR MASONRY FOOTING-COURSES,
IN TERMS OF THE THICKNESS OF THE COURSE, USING 10 AS A FACTOR OF SAFETY.**

| Kind of Stone. | <i>R</i> in Lbs. per Sq. In | Offsets for a Pressure, in Pounds per Square Foot, on the Bottom of the Course of Ma- sonry. | | |
|--|-----------------------------------|--|------|------|
| | | 2000 | 8000 | 4000 |
| Bluestone flagging..... | 2700 | 2.6 | 2.1 | 1.8 |
| Granite..... | 1800 | 2.1 | 1.7 | 1.5 |
| Limestone..... | 1500 | 1.9 | 1.6 | 1.3 |
| Sandstone..... | 1200 | 1.7 | 1.4 | 1.2 |
| Slate..... | 5400 | 3.6 | 2.9 | 2.5 |
| Best hard brick..... | 1500 | 1.9 | 1.6 | 1.3 |
| Hard brick..... | 800 | 1.4 | 1.1 | 1.0 |
| Portland cement concrete, 1:3:6, 1 month old..... | 200 | 0.7 | 0.6 | 0.5 |

In case the earth has sufficient water to keep the foundation damp, a very excellent foundation upon soft earth is a platform of timber composed of heavy sticks laid close together in layers, every alternate layer being right-angled with that adjacent, and thoroughly driftbolted together. Another method is to form a grillage of the timbers and fill the spaces around the sticks with concrete.

In dry soft earth the timber platform may be replaced by a bed of concrete, which is more durable, but not as elastic. The combination of iron or steel beams with concrete has been successfully employed for foundations upon soft earth in Chicago.

The safe projections for timber platforms and those made of steel beams surrounded by concrete (see Fig. 37) can be found from the following formulas.

Let I = the moment of inertia of the section;

h = the thickness of the wooden platform or the depth of the steel beam;

G' = the total load transmitted to a slice of the wooden platform, 12 inches wide, or to one steel beam;

R = the safe fiber stress;

O = the offset, assumed to be the same on opposite sides of the wall supported.

Then, for wooden platforms (12 inches wide)

$$O = \frac{8RI}{G'h} = \frac{8Rh^2}{G'},$$

and for those made of steel beams surrounded by concrete (neglecting the strength of the concrete),

$$O = \frac{8RI}{G'h} = \frac{4RS}{G'},$$

where S is the section modulus given in the various pocket-books published by manufacturers of steel shapes.

If reinforced concrete platforms are used, the safe offsets can be found by following the methods explained for the designing of reinforced concrete walls.

In case the pressure upon the base of the foundation is not uniform, the method outlined for masonry offsets can be applied, using the *maximum* intensity for p_0 in the formulas. (Approximate.)

Pile Foundation.—Pile foundations are employed in all kinds of earth, sometimes to save expense and sometimes because nothing else appears to be as good. In localities where the earth is uncertain in its character the use of piles enables the engineer to put in a foundation which he feels sure is safe, as a single pile thirty feet long will support several tons even when driven into mud, the load in this case being carried almost entirely by the friction of the

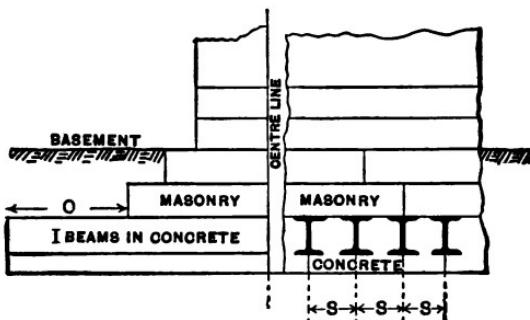


FIG. 37.

mud upon the surface of the pile. If the pile is driven through the mud to a solid stratum below, then the pile acts as a column more or less supported its entire length, and consequently able to carry a very great load.

Piles are usually spaced about three feet on centres, and the tops firmly bedded in a layer of concrete or stayed by a grillage of timber or by a combination of these methods, the object being to thoroughly and evenly distribute the load to be supported.

The supporting power of a pile in a given earth can be found in the following manner:

Let G' = the total load to be supported by the pile, including the weight of the pile;

p_0 = the intensity of the pressure upon the bottom of the pile;

A = the superficial area of the pile in contact with the earth;

and f = a factor depending upon the friction resistance of a unit area of the surface of the pile.

Then for a pile having a diameter of d

$$* G' = \frac{\pi d^2}{4} p_0 + fA.$$

But

$$p_0 = x' \gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \quad \text{and} \quad A = \pi d x'.$$

$$\therefore x' = \frac{G'}{\gamma \frac{\pi d^2}{4} \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 + f \pi d}.$$

For practical purposes this may be written

$$x' = \frac{G'}{\gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 + 3f}.$$

For convenience this may be further simplified for special cases.

* This formula was suggested by reading W. M. Patton's article on piles in his "Practical Foundations."

The following values of f have been given by W. M. Patton, based upon his own and the experience of others:

In very soft silt or liquid mud, $f = 150$ pounds per sq. ft.

In ordinary clay or earth (dry), $f = 300$ " " " "

" " " " (wet), $f = 150$ " " " "

In compact hard clay, $f = 300$ " " " "

In sand, or sand and gravel, $f = 500$ " " " "

For the silt of swamps, muds, etc., ϕ is very nearly if not quite zero. So as to be on the side of safety, ϕ will be taken as zero, $f = 150$ pounds. Then

$$x' = \frac{G'}{120 + 450} = \frac{G'}{570}, \text{ say } \frac{G'}{600},$$

a very simple formula.

For moist clay, $\phi =$ about 17° , $\gamma = 120$ pounds, and $f = 150$ pounds. Then

$$x' = \frac{G'}{120 \cdot 3\frac{1}{3} + 450} = \frac{G'}{850}.$$

For dry, compact sand, $\phi = 27^\circ$, $\gamma = 106$ pounds, and $f = 500$ pounds. Then

$$x' = \frac{G'}{107.7 + 1500} = \frac{G'}{2249}, \text{ say } \frac{G'}{2300}.$$

In a similar manner the safe load for a pile in any earth can be determined when ϕ and f are known. These quantities must be the result of experiment. Any formula which does not include these factors is incomplete, and neglects the factors upon which the supporting power of the pile directly depends.

The character of the earth through which the pile is to be driven can be determined by borings, and thus ϕ and γ determined upon.

The value of f can be found by studying the behavior of

piles already driven in similar earth. Thus it appears that the above formula must be as accurate in results and as safe in application as the majority of the formulas used by engineers in proportioning structures.

The formula is independent of the means by which the pile is driven, as ought to be the case, since very often piles are sunk by water-jets, or even by working them backwards and forward, making the formulas depending upon the weight of a driving-hammer, its fall, and the penetration of the pile during the last few blows useless. Two of the most simple of the many formulas of this class are those of Trautwine and the *Engineering News*, viz. :

$$G' = \frac{\sqrt[3]{h} \times W \times 0.023}{2(1+a)} \text{ (Trautwine); } G' = \frac{2W'h}{a+1} \text{ (Eng. News);}$$

where W' = the weight of hammer in tons; G' = the safe load in tons; W = the weight of the hammer in pounds; h = the fall of the hammer in feet; a = the average penetration of the pile in inches during the last few blows.

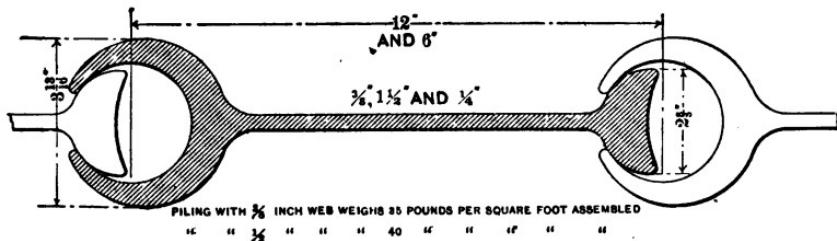
Screw-pile.—Screw-piles are usually round, and have at the bottom a cast or wrought iron screw. The piles are of wood, cast iron, or wrought iron. The diameter of the screw is from two to eight times the diameter of the pile, and its pitch from one fourth to one half its diameter. The screw seldom has but one turn. The piles are sunk by turning them by means of levers or by power. (Fig. 45.)

The load which the pile will carry depends principally upon the supporting power of the earth at the depth of the screw and the area of the screw, though in all cases there is more or less frictional resistance upon the surface of the pile proper. If x' is the depth of the screw and p , the

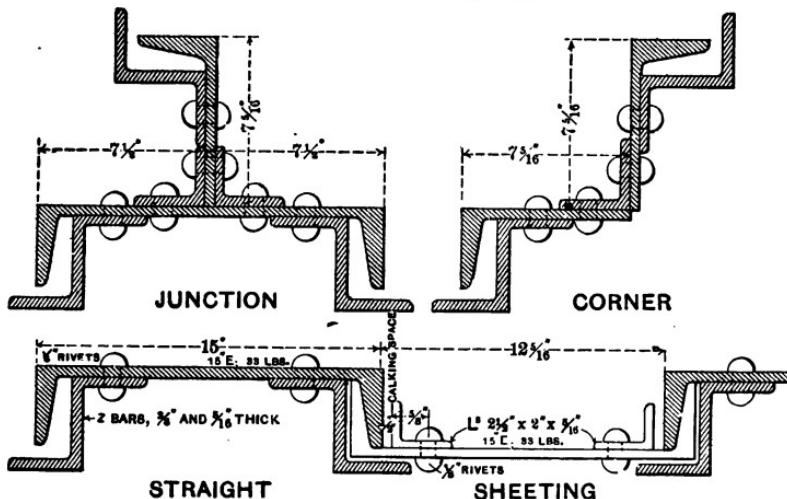
allowable intensity of the pressure upon the earth at that depth, then

$$p_0 = x' \gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \text{ (page 39).}$$

Screw-piles can be advantageously employed for supporting structures above water where the upper ends of the



United States Sheet-piling.



The Friestedt Interlocking Sheet-piling.

piles can be used as columns. They are chiefly employed in light-house construction.

Sheet-piles.—Sheet-piles are usually of wood in the form of planks, and are driven as closely together, edge to edge, as possible, the object being to form a water-tight barrier.

To make the joints tight the planks are oftened tongued and grooved. A patent sheet-pile is formed by bolting together three planks of equal width, so that the middle plank will form the tongue on one side and the outside planks the groove on the other side. Sheet-piles are also employed to confine soft earths.

Several forms of steel piling have recently come into general use. They are in many ways superior to wooden piles and often less expensive in the long run.

FOUNDATIONS UNDER WATER AND DEEP FOUNDATIONS.

Foundations under water differ in general but little from those upon dry earth, the effect of water, ice, etc., upon the structure, however, constitute additional problems to be solved for each locality.

A few of the various methods employed in placing foundations under water or at great depths will be very briefly described.

Coffer-dams.—A coffer-dam is merely a tight wall surrounding the locality where the foundation is to be placed, excluding water from the enclosure, which can be pumped dry and the surface prepared to receive the foundation.

In quiet and shallow water the dam may be made of earth; or sheet-piles banked with earth.

In deep water large piles are driven every few feet in two rows around the site, to which horizontal timbers are bolted, acting as guides and supports to a double row of sheet-piles, between which is placed puddled earth. To prevent bending, the large piles are cross-tied with bolts.

The space enclosed should be somewhat larger than required by the foundation, to allow room for materials, etc. (Fig. 46.)

Timber Cribs.—A timber crib is a box built of large timbers and divided into cells by cross partitions. The joints and splices of the timbers employed are arranged so that walls and partitions are thoroughly tied together. In

case a tight wall-crib is wanted the timbers may be dapped one fourth their depth on both sides or halved together. Cribs are built in the shape best suited to the purpose for which they are to be used. They are usually constructed at some convenient point near the site of the foundation, and then towed to the place where they are to be sunk. In constructing the crib a few of the cells are planked near the bottom. These are filled with stone until the crib sinks to the surface previously prepared to receive it. The other cells are now filled with stone and the regular masonry commenced. Sometimes the top of the crib is planked over before the masonry is started. (Fig. 44.)

The surface which is to receive the crib may be soft mud, riprap, rock, or piles. The crib is allowed to sink into the mud and to rest upon riprap which has been levelled. If the surface is level rock, the crib is merely sunk; but if the rock is uneven, it is either levelled or the crib is sunk until it just touches rock at some point, when riprap is thrown around and under the crib.

Timber cribs are extensively employed in various classes of engineering works for both temporary and permanent structures.

In permanent structures the timbers supporting masonry, etc., should always be under water.

Timber cribs are sometimes used as coffer-dams by making the outside cells water-tight. The crib is sunk into the mud, or the bottom edges banked with earth, etc., until the interior can be kept dry by pumping.

Open Caissons.—An open caisson is a strong water-tight box which is floated to the site of the foundation and sunk to its place by the masonry proper, which is built inside the box. After the bottom has reached its position and the top of the masonry is above water, the sides are removed,

leaving the bottom of the box as a platform supporting the masonry. The surface to receive an open caisson is prepared by dredging, throwing in riprap, driving piles, etc., as best suits the locality. (Fig. 47.)

Cushing Cylinder Piers.—A cluster of piles is first driven as closely together as possible, and their tops thoroughly bolted one to the other. Then an iron cylinder is placed around the cluster and built up in sections until the top is above water. Then the cylinder is made to sink by dredging out the material inside by water-jets, by disturbing the material around the edges, etc., until a desired depth is reached, sections being bolted to the top of the cylinder as needed. The cylinder is now filled with concrete to the top and covered with an iron cap which receives the load to be carried. The size and number of cylinders employed depends upon the superstructure.

For ordinary bridges two cylinders cross-braced form a pier.

The supporting power depends upon the piles principally, though the friction upon the outside of the cylinders offers some resistance to settlement.

Pneumatic Caissons.—A pneumatic caisson is essentially an air-tight box with the open side imbedded in earth, from which the air is pumped to allow the box to sink or into which air is pumped to prevent sinking. In water the caisson usually carries a water-tight timber crib, which in turn supports a timber coffer-dam, the crib enabling the structure to be loaded with stone according to the requirements of the sinking operation, and the coffer-dam keeping the water out near the surface. Various combinations of caisson, crib, and coffer-dam are made, however, to suit conditions. (Fig. 48.)

The ordinary method of sinking caissons is to pump in

enough air to exclude water from the chamber, while laborers dig out the material over the surface and near the edges of the chamber, this material being removed by various methods such as pumps, lifts, etc. When sufficient material has been removed, all the laborers leave the caisson, leaving one man only who watches for leaks; the air-pressure is then lowered a little, and the caisson with its superstructure sinks. This process is repeated until a solid foundation is reached, when the caisson is filled with concrete, as also are the cribs, etc., if any, above the caisson.

TYPES OF EXISTING FOUNDATIONS.

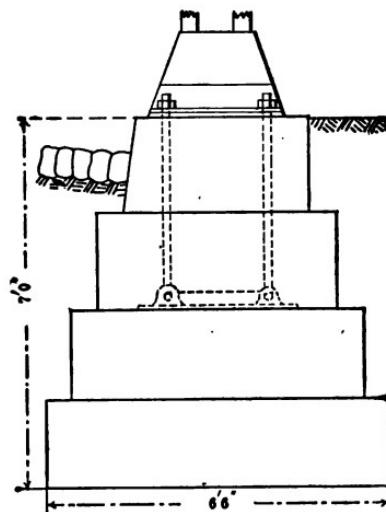


FIG. 38.

Concrete Pier used as Foundation for Elevated Railroad Columns
(*Engineering and Building Record*, Sept. 14, 1895.)

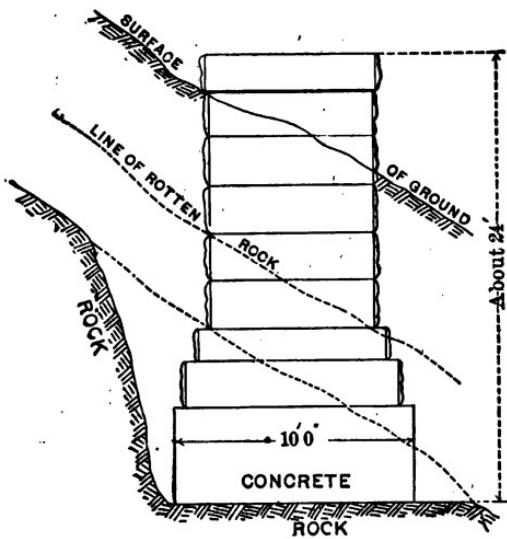


FIG. 39.

Elevation of Masonry Pier with Bottom Course of Concrete. Illustrating the removal of rotten rock and the levelling of the rock surface. (Marent Galch Viaduct, N. P. R. R.; *Trans. Am. Soc. C. E.*, Sept., 1891.)

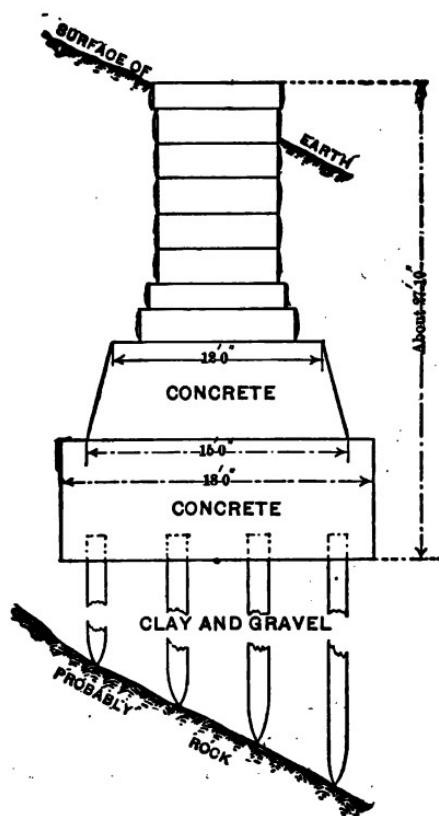


FIG. 40.

Elevation of another Pier of the Marent Viaduct Foundations. Showing the application of piles and concrete to obtain a solid foundation.

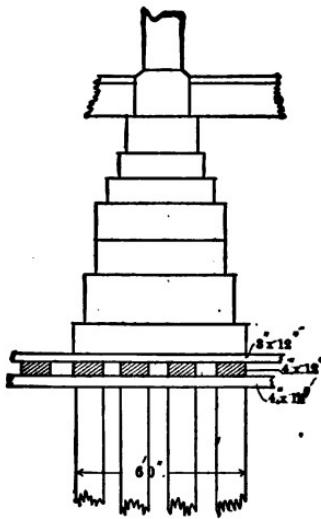


FIG. 41.

Elevation of a Pier in the Foundation of a Chicago Grain Elevator. Illustrating the use of piles and a wooden platform in soft ground. Piles are from 20 to 40 feet long, and reach hardpan. Twelve piles are placed under each post, and each pile supports a load of about 22 tons. (*Engineering and Building Record*, Nov. 12, 1895.)

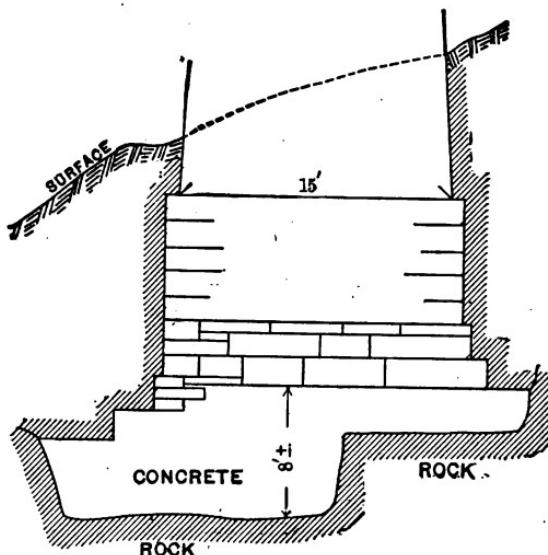


FIG. 42.

End Elevation of Masonry Pier supporting Stone Arches of Washington Bridge. Illustrating the use of concrete to level the rock surface to receive masonry.

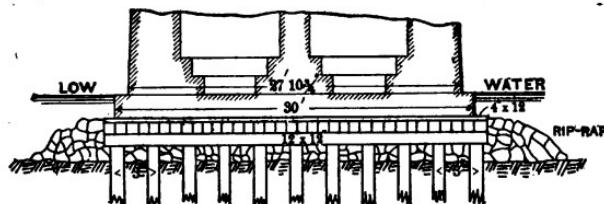


FIG. 43.

Section through Centre of Foundation of Pivot Pier of Grand Forks Bridge. Illustrating the use of piles, wooden platform, and riprap. (Baker.)

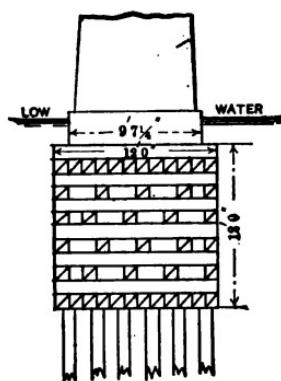


FIG. 44.

End Elevation of Foundation of Pier of Croix River Bridge. Illustrating the use of timber crib and piles. (*Baker.*)

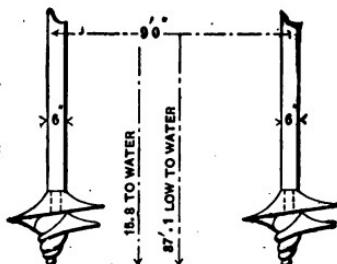


FIG. 45.

Mobile River Bridge Piers. Composed of two rows of screw-piles, about 9 feet centre to centre, with piles spaced about 8 feet apart. (See *Engineering News*, vol. xiii. p. 210.)

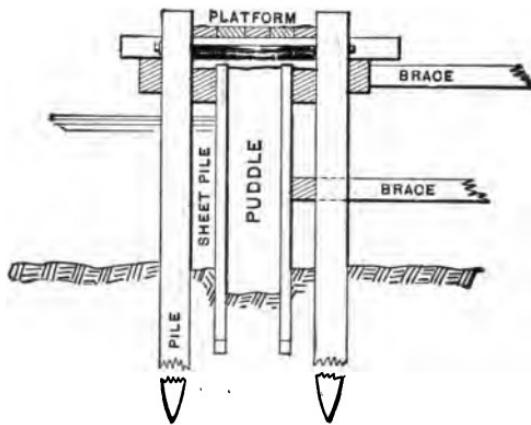


FIG. 46.

Sketch showing Cross-section of Cofferdam.

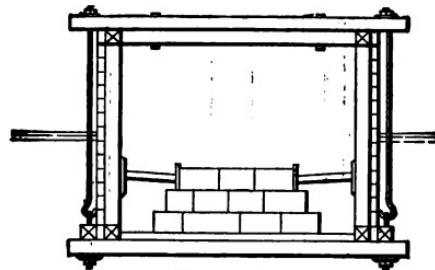


FIG. 47.

Sketch showing Essential Features of Open Caisson.

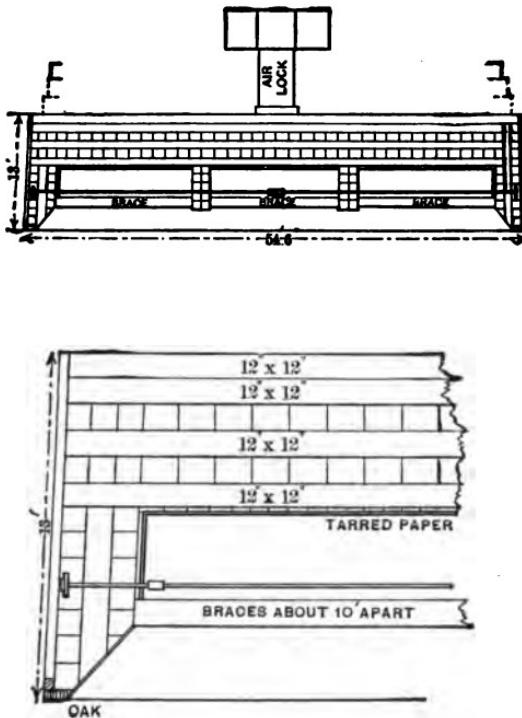
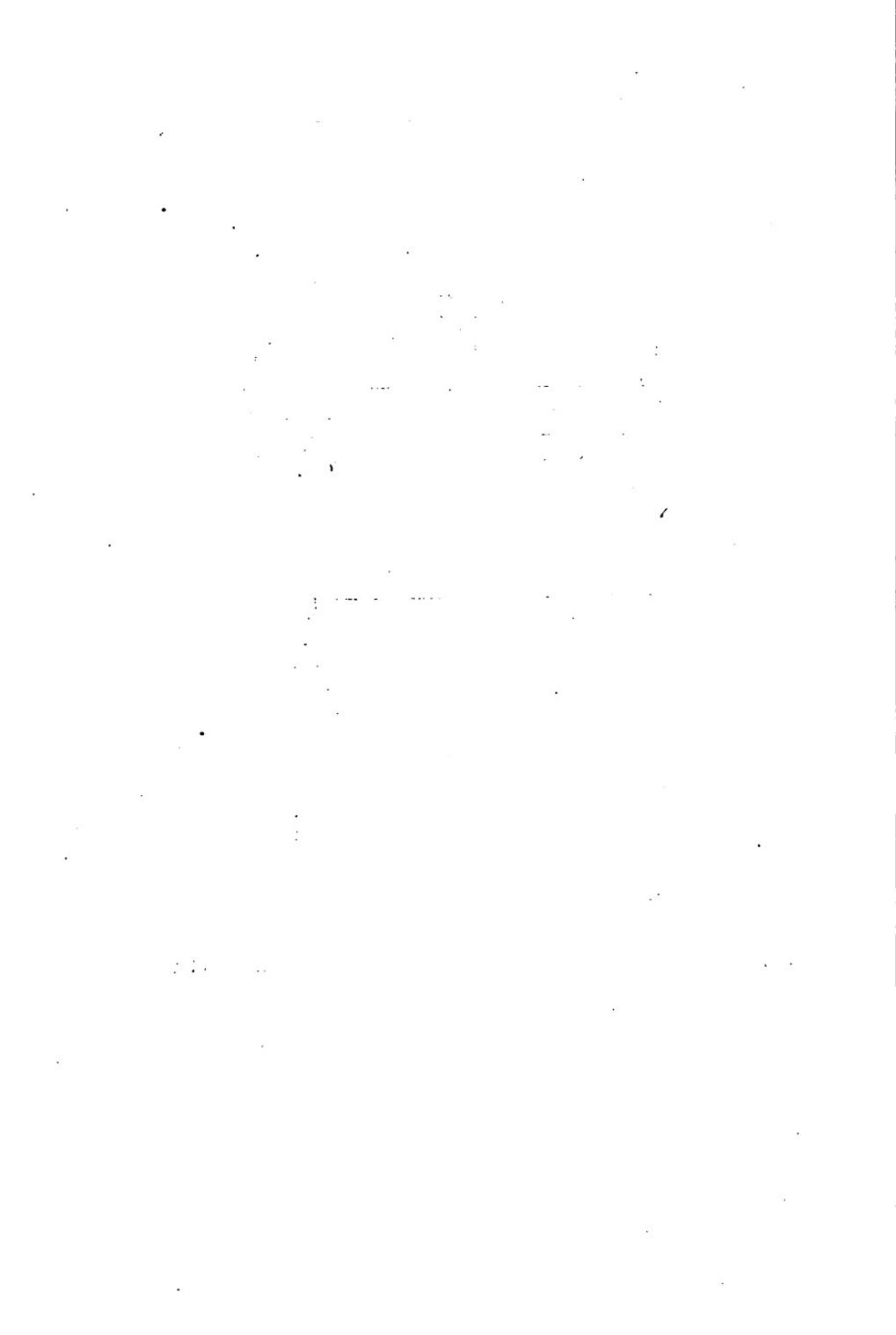


FIG. 48.

Section of One of the Caissons employed in the Foundations of the Piers for the Washington Bridge.



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|------------|---------------------|----------------|
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| Andé, | Holzhey, | Prony, |
| Andoy, | de Lafont, | Rankine, |
| Belidor, | Levi, | Rebhann, |
| Blaveau, | de Köszegh Martony, | Rondelet, |
| Bullet, | Maschek, | Saint-Guilhem, |
| Considère, | Mayniel, | Saint-Venant, |
| Coulomb, | Mohr, | Sallonnier, |
| Couplet, | Montlong, | Scheffler, |
| Culmann, | Moseley, | Trincaux, |
| Français, | Navier, | Vauban, |
| Gadroy, | Ortmann, | Winkler, |
| Gauthey, | v. Ott, | Woltmann. |
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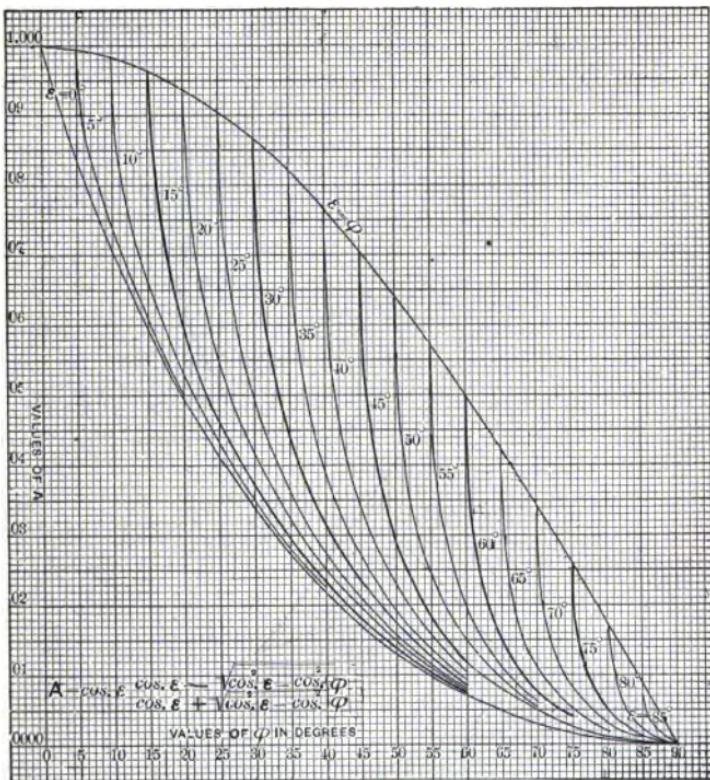
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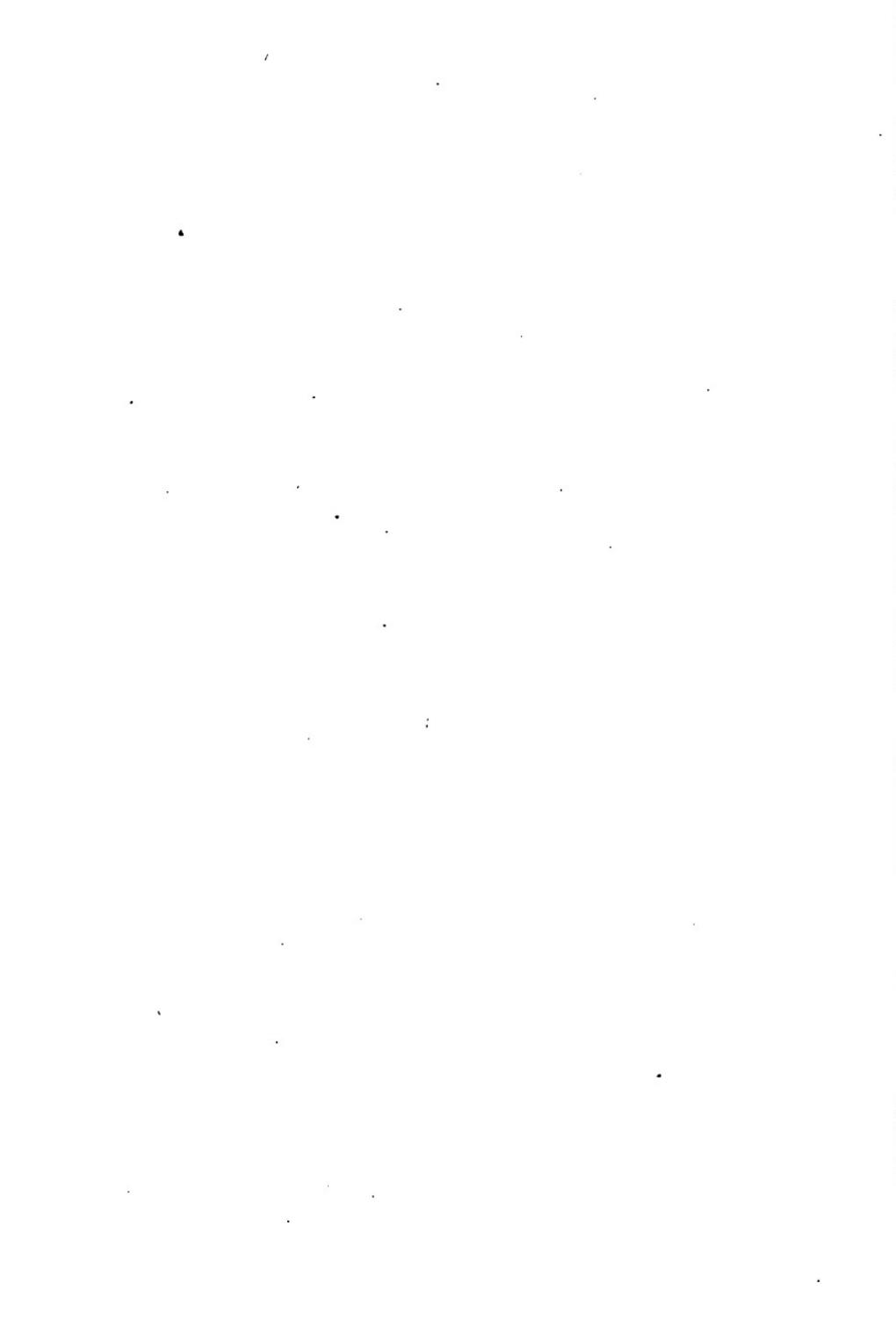
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DIAGRAM I.





TABLES.

Table I contains the crushing-strengths and the average weights of stone likely to be used in the construction of retaining-walls and foundations; also the average weights of different earths.

Table II contains the coefficients of friction, limiting angles of friction, and the reciprocals of the coefficients of friction for various substances.

Tables III, IV, and V contain the values of the coefficients [see equation (1')] (B), (C), (D) and (E), where

$$(B) = \frac{\cos(\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon}, \quad (C) = \sin^2 \alpha, \quad (D) = \left\{ \frac{\cos(\epsilon - \alpha)}{\cos \epsilon} \right\}^2$$

and $(E) = 2 \sin \alpha \sin \epsilon \frac{\cos(\epsilon - \alpha)}{\cos \epsilon}.$

The tables were computed with a Thacher calculating instrument and checked by means of diagrams. It is believed that they are correct to the second place of decimals; an error in the third place of decimals does not affect the results for practical purposes.

Table VI contains the natural sines, cosines and tangents.

TABLE I.

* VALUES OF *W* FOR MASONRY.

| Name. | Crushing Strength in Tons per Sq. Ft., Safe. | Average Weight, Lbs. per Cu. Ft. |
|--|---|---|
| <i>Brick.</i> | | |
| Pressed brick, fine joints..... | 6-15 | 140 |
| Medium-quality brick..... | 6-10 | 125 |
| Coarse, inferior, soft..... | 5-7 | 100 |
| <i>Granite.</i> | | |
| Well dressed..... | 60+ | 165 |
| Well scabbled mortar rubble, $\frac{1}{2}$ mass mortar..... | 20-40 | 154 |
| “ “ dry rubble..... | 20-40 | 138 |
| Roughly scabbled mortar rubble, $\frac{1}{4}$ - $\frac{1}{2}$ mass mortar..... | 20-40 | 150 |
| Scabbled dry rubble..... | 5-16 | 125 |
| <i>Limestone.</i> | | |
| Ashlar, largest stones, thinnest joints..... | 40+ | 160 |
| Ashlar, 12"-20" courses, $\frac{3}{8}"$ - $\frac{1}{2}"$ joints..... | 40- | 155 |
| Squared stone..... | 10-20 | 148 |
| Best rubble..... | | 142 |
| Rough rubble..... | | 136 |
| <i>Sandstone.</i> | | |
| Ashlar, large stones, thin joints..... | 30+ | 138 |
| Ashlar 12"-20" courses, $\frac{3}{8}"$ - $\frac{1}{2}"$ joints..... | 30- | 133 |
| Squared stone..... | 10-20 | 127 |
| Best rubble..... | 2 $\frac{1}{2}$ -5? | 122 |
| Rough rubble..... | 2 $\frac{1}{2}$ -5? | 117 |

* Based on data given in Architects and Builders' Pocket-book, by F. E. Kidder, and Trautwine's Engineers' Pocket-book. (John Wiley & Sons.)

TABLE I—*Continued.*

THE ULTIMATE CRUSHING STRENGTH IN POUNDS PER SQUARE INCH OF
PORTLAND CONCRETE ACCORDING TO THACHER'S FORMULAS BASED
ON WATERTOWN EXPERIMENTS.

| Mixture. | Age in Days. | | | | Remarks. |
|-----------------------|----------------|---------------|------------------|----------------|--|
| | Seven Days. | One Month. | Three Months. | Six Months. | |
| 1: 1 : 3 | 1600 | 2750 | 3360 | 4300 | $S = 1800 - 200x$; 7 days. |
| 1: 2 : 4 | 1400 | 2400 | 2900 | 3700 | $S = 3100 - 350x$; 1 month |
| 1: $2\frac{1}{2}$: 5 | 1300 | 2225 | 2670 | 3400 | $S = 3820 - 460x$; 3 months. |
| 1: 3 : 6 | 1200 | 2050 | 2440 | 3100 | $S = 4900 - 600x$; 8 " |
| 1: $3\frac{1}{2}$: 7 | 1100 | 1875 | 2210 | 2800 | |
| 1: 4 : 8 | 1000 | 1700 | 1980 | 2500 | x = parts of sand to one part cement; |
| 1: 5 : 10 | 800 | 1350 | 1520 | 1900 | |
| 1: 6 : 12 | 600 | 1000 | 1060 | 1300 | S = ultimate strength for 12-inch cubes. |

See *Cement*, May, 1902.

Ordinarily the tensile strength of concrete may be assumed as *one tenth* the compressive strength.

The weight of concrete varies from 105 to 150 pounds per cubic foot.

| | | | |
|------------------------------|---|------------------------|-----------------------------------|
| According to H. G. Richey | { | Cinder concrete | weighs about 105 lbs. per cu. ft. |
| | | Slag concrete | " " 135 " " " |
| | | Crushed-stone concrete | " " 140 " " " |
| | | Gravel concrete | " " 150 " " " |

TABLE I—Continued.

CRUSHING STRENGTH OF BRICK AND STONE IN POUNDS PER SQUARE INCH
 (NEAREST HUNDRED POUNDS).
 (Frank E. Kidder.)

The values given are for stones tested upon their natural beds.

| | | |
|---------------------------------|--------------------------------|-------------|
| Brick. | Common, Massachusetts..... | 10000 |
| | “ St. Louis..... | 6400 |
| | “ Washington, D. C..... | 7400 |
| | Paving, Illinois..... | 6000-13000 |
| Granite. | Blue, Fox Island, Me..... | 14900 |
| | Gray, Vinal Haven, Me..... | 13000-18000 |
| | Westerly, R. I..... | 15000 |
| | Rockport and Quincy, Mass..... | 17800 |
| | Milford, Conn..... | 22600 |
| | Staten Island, N. Y..... | 22300 |
| | East St. Cloud, Minn..... | 28000 |
| | Gunnison, Colo..... | 13000 |
| | * Red, Platte Cañon, Colo..... | 14600 |
| | * Bradford, Conn..... | 15700 |
| | * Troy, N. H..... | 26200 |
| | * Pigeon Hill, Mass..... | 19700 |
| Limestone. | Glens Falls, N. Y..... | 11500 |
| | Joliet, Ill..... | 12800 |
| | Bedford, Ind..... | 6000-10000 |
| | Salem, Ind..... | 8600 |
| | Red Wing, Minn..... | 23000 |
| | Stillwater, Minn..... | 10800 |
| | Rutland, Vt., marble..... | 10700 |
| | * Creole marble, Ga..... | 13500 |
| | * Cherokee marble, Ga..... | 12600 |
| | * Etowah marble, Ga..... | 14100 |
| | * Kennesaw marble, Ga..... | 9600 |
| | * Marble Hill marble, Ga..... | 11500 |
| 146-180 lbs. per cubic foot. | * Tuckahoe marble, N. Y..... | 16200 |
| | * Mt. Vernon marble, Ky..... | 7600 |

* From tests made at Watertown, Mass., and given in Johnson's Materials of Construction

TABLE I—*Continued.*

CRUSHING STRENGTH OF BRICK AND STONE IN POUNDS PER SQUARE INCH
(NEAREST HUNDRED POUNDS).
(Frank E. Kidder.)

| | | |
|--|---|-------------|
| Sandstone. 127-151 lbs. per cubic foot. | Brown, Dorchester, N. B. | 9200 |
| | Mary's Point, N. B., fine grain, dark brown. | 7700 |
| | Conn. brownstone. | 7000-13000 |
| | Longmeadow, Mass., reddish brown. | 7000-14000 |
| | " " average.... | 12000 |
| | Little Falls, N. Y. | 9900 |
| | Medina, N. Y. | 17000 |
| | Potsdam, N. Y. | 18000-42000 |
| | Cleveland, Ohio. | 6800 |
| | N. Amherst, Ohio. | 6200 |
| | Berea, Ohio. | 8000-10000 |
| | Hummelstown, Pa. | 12800 |
| | Fond du Lac, Minn. | 8800 |
| | " " " Wis. | 6200 |
| | Manitou, Colo., light red. | 6000-11000 |
| | St. Vrain, Colo., hard, laminated. | 11500 |
| | * Cooper, Oregon. | 15200 |
| | * Cromwell, Conn. | 10800 |
| | * Maynard, Conn. | 9900 |
| | * Kibble, Mass. | 10400 |
| | * Worcester, Mass. | 9800 |

ALLOWABLE VALUES OF *R* FOR VARIOUS MATERIALS IN POUNDS
PER SQUARE INCH.

| | | | |
|-------------------------------|------------------------------------|---|------|
| Steel..... | 16000 | Granite..... | 180* |
| Cast iron ... | { 2500 tension 8000 compression | Limestone..... | 150* |
| Spruce and white pine... 1000 | | Sandstone..... | 120* |
| White oak..... 1250 | | Slate..... | 540* |
| L. L. Southern pine..... 1500 | | Best hard brick..... | 150* |
| Bluestone flagging..... 270* | | Hard brick..... | 80* |
| | | Portland cement concrete, 1:3:6, one month old.. | 20* |

* Safety factor = 10 for tension.

TABLE I—Continued.

VALUES OF γ (FOR THE MOST PART FROM "CAMBRIA STEEL").

| Name. | Approximate Values of ϕ . | Average Weight in Lbs. per Cu. Ft. |
|---|--------------------------------|------------------------------------|
| Anthracite, broken, of any size, loose..... | 27° | 52-56 |
| " " moderately shaken..... | ... | 56-60 |
| " solid..... | ... | 93.5 |
| Ashes of soft coal, solidly packed..... | ... | 40-45 |
| Cement, hydraulic, Am. ground, loose..... | ... | 56 |
| " " Cumberland, loose..... | ... | 65 |
| " " thoroughly shaken..... | ... | 85 |
| " English Portland..... | ... | 81-102 |
| " American Portland, loose..... | ... | 88 |
| " " " thoroughly shaken..... | 15° | 110 |
| Coal, bituminous, solid..... | ... | 84 |
| " " broken, of any size, loose..... | 35° | 47-52 |
| " " " moderately shaken..... | ... | 51-56 |
| Coke, loose, good quality..... | ... | 23-32 |
| Earth, common loam, dry, loose..... | ... | 72-80 |
| " " " shaken..... | 40° | 82-92 |
| " " " rammed..... | ... | 90-100 |
| " " " slightly moist, loose..... | 45° | 70-76 |
| " " " more moist, loose..... | ... | 66-68 |
| " " " " sha en..... | ... | 75-90 |
| " " " " packed..... | ... | 90-100 |
| " " " as soft flowing mud..... | ... | 104-112 |
| The same well pressed..... | ... | 110-120 |
| Gravel, about the same as sand..... | ... | ... |
| Mud, dry, close..... | ... | 80-110 |
| " wet, moderately pressed..... | ... | 110-130 |
| " " fluid..... | ... | 104-120 |
| Petroleum..... | 0° | 54.8 |
| Salt, coarse..... | ... | 45 |
| Sand of pure quartz, dry, loose..... | 35° | 90-106 |
| " " " voids full of water..... | 30° | 118-129 |
| " " " very large grains..... | 35°(?) | 117 |
| Water, pure..... | 0° | 62.417 |

TABLE II.

* ANGLES AND COEFFICIENTS OF FRICTION.

| | $\tan \phi$. | ϕ | $\frac{1}{\tan \phi}$ |
|--|---------------|-------------|-----------------------|
| Dry masonry and brickwork | 0.6 to 0.7 | 31° to 35° | 1.67 to 1.48 |
| Masonry and brickwork with damp mortar..... | 0.74 | 36½° | 1.35 |
| Timber on stone..... | about 0.4 | 22° | 2.5 |
| Iron on stone | 0.7 to 0.8 | 35° to 16½° | 1.43 to 3.38 |
| Timber on timber..... | 0.5 " 0.2 | 26½° " 11½° | 2 " 5 |
| Timber on metals..... | 0.6 " 0.2 | 31° " 11½° | 1.67 " 5 |
| Metals on metals..... | 0.25 " 0.15 | 14° " 8½° | 4 " 6.67 |
| Masonry on dry clay..... | 0.51 | 27° | 1.96 |
| " " moist clay..... | 0.38 | 18½° | 3. |
| Earth on earth | 0.25 to 1.0 | 14° to 45° | 4 to 1 |
| Earth on earth, dry sand, clay, and mixed earth.... | 0.38 " 0.75 | 21° " 37° | 2.68 " 1.38 |
| Earth on earth, damp clay. | 1.0 | 45° | 1 |
| Earth on earth, wet clay. | 0.81 | 17° | 3.23 |
| Earth on earth, shingle and gravel..... | 0.81 | 39° to 48° | 1.28 to 0.9 |

* From Rankine's Applied Mechanics.

TABLES.

TABLE III.

| ϵ | $\alpha = 5^\circ$ (B) | $\alpha = 6^\circ$ (B) | $\alpha = 7^\circ$ (B) | $\alpha = 8^\circ$ (B) | $\alpha = 9^\circ$ (B) |
|------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 0 | 1.004 | 1.005 | 1.007 | 1.010 | 1.012 |
| 5 | 1.012 | 1.015 | 1.018 | 1.022 | 1.026 |
| 10 | 1.019 | 1.024 | 1.029 | 1.035 | 1.040 |
| 15 | 1.027 | 1.034 | 1.041 | 1.048 | 1.055 |
| 20 | 1.036 | 1.044 | 1.052 | 1.062 | 1.071 |
| 25 | 1.045 | 1.055 | 1.065 | 1.076 | 1.088 |
| 30 | 1.055 | 1.066 | 1.079 | 1.092 | 1.105 |
| 35 | 1.065 | 1.079 | 1.094 | 1.109 | 1.124 |
| 40 | 1.078 | 1.094 | 1.111 | 1.129 | 1.147 |
| 45 | 1.098 | 1.111 | 1.131 | 1.152 | 1.173 |
| | (C) | (C) | (C) | (C) | (C) |
| | 0.008 | 0.011 | 0.015 | 0.019 | 0.024 |

TABLE IV.

| ϵ | $\alpha = 5^\circ$ (D) | $\alpha = 6^\circ$ (D) | $\alpha = 7^\circ$ (D) | $\alpha = 8^\circ$ (D) | $\alpha = 9^\circ$ (D) |
|------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 0 | 0.992 | 0.989 | 0.985 | 0.981 | 0.976 |
| 5 | 1.008 | 1.008 | 1.006 | 1.005 | 1.008 |
| 10 | 1.023 | 1.026 | 1.028 | 1.030 | 1.031 |
| 15 | 1.040 | 1.046 | 1.051 | 1.056 | 1.060 |
| 20 | 1.057 | 1.066 | 1.075 | 1.084 | 1.092 |
| 25 | 1.075 | 1.089 | 1.102 | 1.114 | 1.125 |
| 30 | 1.096 | 1.113 | 1.130 | 1.147 | 1.163 |
| 35 | 1.118 | 1.140 | 1.164 | 1.183 | 1.204 |
| 40 | 1.144 | 1.172 | 1.199 | 1.226 | 1.253 |
| 45 | 1.174 | 1.208 | 1.242 | 1.276 | 1.309 |

TABLE V.

| ϵ | $\alpha = 5^\circ$ (E) | $\alpha = 6^\circ$ (E) | $\alpha = 7^\circ$ (E) | $\alpha = 8^\circ$ (E) | $\alpha = 9^\circ$ (E) |
|------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0.015 | 0.018 | 0.021 | 0.024 | 0.027 |
| 10 | 0.031 | 0.037 | 0.048 | 0.049 | 0.055 |
| 15 | 0.046 | 0.055 | 0.065 | 0.074 | 0.083 |
| 20 | 0.061 | 0.074 | 0.086 | 0.099 | 0.113 |
| 25 | 0.076 | 0.092 | 0.108 | 0.124 | 0.140 |
| 30 | 0.091 | 0.110 | 0.130 | 0.149 | 0.169 |
| 35 | 0.106 | 0.128 | 0.151 | 0.174 | 0.197 |
| 40 | 0.120 | 0.145 | 0.172 | 0.198 | 0.225 |
| 45 | 0.134 | 0.162 | 0.192 | 0.222 | 0.253 |

TABLE III—Continued.

| • | $\alpha = 10^\circ$ | $\alpha = 11^\circ$ | $\alpha = 12^\circ$ | $\alpha = 13^\circ$ | $\alpha = 14^\circ$ |
|----|---------------------|---------------------|---------------------|---------------------|---------------------|
| | (B) | (B) | (B) | (B) | (B) |
| 0 | 1.015 | 1.019 | 1.022 | 1.026 | 1.031 |
| 5 | 1.031 | 1.037 | 1.041 | 1.047 | 1.053 |
| 10 | 1.046 | 1.055 | 1.061 | 1.068 | 1.076 |
| 15 | 1.063 | 1.073 | 1.081 | 1.090 | 1.100 |
| 20 | 1.081 | 1.092 | 1.103 | 1.112 | 1.125 |
| 25 | 1.099 | 1.112 | 1.124 | 1.136 | 1.150 |
| 30 | 1.119 | 1.135 | 1.151 | 1.163 | 1.179 |
| 35 | 1.141 | 1.159 | 1.175 | 1.195 | 1.211 |
| 40 | 1.166 | 1.186 | 1.205 | 1.225 | 1.245 |
| 45 | 1.195 | 1.218 | 1.240 | 1.263 | 1.288 |
| | (C) | (C) | (C) | (C) | (C) |
| | 0.030 | 0.036 | 0.048 | 0.051 | 0.029 |

TABLE IV—Continued.

| • | $\alpha = 10^\circ$ | $\alpha = 11^\circ$ | $\alpha = 12^\circ$ | $\alpha = 13^\circ$ | $\alpha = 14^\circ$ |
|----|---------------------|---------------------|---------------------|---------------------|---------------------|
| | (D) | (D) | (D) | (D) | (D) |
| 0 | 0.970 | 0.964 | 0.957 | 0.950 | 0.943 |
| 5 | 1.000 | 0.997 | 0.993 | 0.988 | 0.983 |
| 10 | 1.031 | 1.031 | 1.030 | 1.028 | 1.026 |
| 15 | 1.064 | 1.067 | 1.069 | 1.061 | 1.072 |
| 20 | 1.099 | 1.105 | 1.110 | 1.116 | 1.121 |
| 25 | 1.136 | 1.147 | 1.156 | 1.165 | 1.173 |
| 30 | 1.178 | 1.194 | 1.204 | 1.220 | 1.232 |
| 35 | 1.224 | 1.244 | 1.262 | 1.281 | 1.300 |
| 40 | 1.291 | 1.304 | 1.328 | 1.358 | 1.377 |
| 45 | 1.342 | 1.375 | 1.407 | 1.438 | 1.469 |

TABLE V—Continued.

| • | $\alpha = 10^\circ$ | $\alpha = 11^\circ$ | $\alpha = 12^\circ$ | $\alpha = 13^\circ$ | $\alpha = 14^\circ$ |
|----|---------------------|---------------------|---------------------|---------------------|---------------------|
| | (E) | (E) | (E) | (E) | (E) |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0.080 | 0.082 | 0.086 | 0.089 | 0.042 |
| 10 | 0.061 | 0.067 | 0.073 | 0.079 | 0.085 |
| 15 | 0.093 | 0.102 | 0.111 | 0.119 | 0.130 |
| 20 | 0.124 | 0.137 | 0.150 | 0.163 | 0.175 |
| 25 | 0.156 | 0.173 | 0.189 | 0.205 | 0.221 |
| 30 | 0.188 | 0.208 | 0.216 | 0.248 | 0.269 |
| 35 | 0.220 | 0.244 | 0.268 | 0.292 | 0.316 |
| 40 | 0.252 | 0.280 | 0.308 | 0.336 | 0.365 |
| 45 | 0.284 | 0.316 | 0.349 | 0.382 | 0.415 |

TABLE III—Continued.

| ϵ | $\alpha = 15^\circ$ | $\alpha = 16^\circ$ | $\alpha = 17^\circ$ | $\alpha = 18^\circ$ | $\alpha = 20^\circ$ |
|------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| | (B) | (B) | (B) | (B) | (B) |
| 0 | 1.035 | 1.040 | 1.048 | 1.051 | 1.062 |
| 5 | 1.059 | 1.066 | 1.076 | 1.081 | 1.098 |
| 10 | 1.084 | 1.093 | 1.104 | 1.112 | 1.132 |
| 15 | 1.110 | 1.120 | 1.134 | 1.138 | 1.168 |
| 20 | 1.135 | 1.149 | 1.165 | 1.177 | 1.218 |
| 25 | 1.165 | 1.179 | 1.197 | 1.211 | 1.245 |
| 30 | 1.195 | 1.212 | 1.233 | 1.248 | 1.288 |
| 35 | 1.229 | 1.249 | 1.272 | 1.291 | 1.339 |
| 40 | 1.268 | 1.291 | 1.317 | 1.340 | 1.389 |
| 45 | 1.313 | 1.338 | 1.369 | 1.393 | 1.451 |
| | (C) | (C) | (C) | (C) | (C) |
| | 0.067 | 0.076 | 0.086 | 0.095 | 0.117 |

TABLE IV—Continued.

| ϵ | $\alpha = 15^\circ$ | $\alpha = 16^\circ$ | $\alpha = 17^\circ$ | $\alpha = 18^\circ$ | $\alpha = 20^\circ$ |
|------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| | (D) | (D) | (D) | (D) | (D) |
| 0 | 0.933 | 0.924 | 0.915 | 0.905 | 0.893 |
| 5 | 0.977 | 0.971 | 0.964 | 0.957 | 0.940 |
| 10 | 1.023 | 1.018 | 1.016 | 1.011 | 1.000 |
| 15 | 1.073 | 1.078 | 1.071 | 1.069 | 1.068 |
| 20 | 1.124 | 1.127 | 1.129 | 1.131 | 1.132 |
| 25 | 1.181 | 1.188 | 1.194 | 1.200 | 1.208 |
| 30 | 1.244 | 1.256 | 1.266 | 1.276 | 1.293 |
| 35 | 1.316 | 1.332 | 1.348 | 1.368 | 1.390 |
| 40 | 1.400 | 1.422 | 1.444 | 1.465 | 1.505 |
| 45 | 1.500 | 1.530 | 1.559 | 1.588 | 1.643 |

TABLE V—Continued.

| ϵ | $\alpha = 15^\circ$ | $\alpha = 16^\circ$ | $\alpha = 17^\circ$ | $\alpha = 18^\circ$ | $\alpha = 20^\circ$ |
|------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| | (E) | (E) | (E) | (E) | (E) |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0.045 | 0.047 | 0.050 | 0.053 | 0.058 |
| 10 | 0.091 | 0.097 | 0.102 | 0.108 | 0.119 |
| 15 | 0.139 | 0.148 | 0.157 | 0.165 | 0.183 |
| 20 | 0.188 | 0.200 | 0.213 | 0.225 | 0.249 |
| 25 | 0.238 | 0.254 | 0.270 | 0.177 | 0.318 |
| 30 | 0.289 | 0.309 | 0.329 | 0.349 | 0.389 |
| 35 | 0.341 | 0.365 | 0.390 | 0.414 | 0.463 |
| 40 | 0.394 | 0.423 | 0.452 | 0.481 | 0.539 |
| 45 | 0.448 | 0.482 | 0.516 | 0.551 | 0.620 |

TABLE VI.

**NATURAL SINES, COSINES, TANGENTS
AND COTANGENTS.**

| | 0° | | 1° | | 2° | | 3° | | 4° | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| | Sine | Cosin | |
| 0 | .00000 | One. | .01745 | .99985 | .03490 | .99939 | .05234 | .99863 | .06976 | .99756 | 60 |
| 1 | .00029 | One. | .01774 | .99964 | .03518 | .99938 | .05263 | .99861 | .07005 | .99754 | 59 |
| 2 | .00058 | One. | .01803 | .99984 | .03548 | .99937 | .05292 | .99860 | .07034 | .99752 | 58 |
| 3 | .00087 | One. | .01832 | .99983 | .03577 | .99936 | .05321 | .99858 | .07063 | .99750 | 57 |
| 4 | .00116 | One. | .01862 | .99983 | .03606 | .99935 | .05350 | .99857 | .07092 | .99748 | 56 |
| 5 | .00145 | One. | .01891 | .99982 | .03635 | .99934 | .05379 | .99855 | .07121 | .99746 | 55 |
| 6 | .00175 | One. | .01920 | .99982 | .03664 | .99933 | .05408 | .99854 | .07150 | .99744 | 54 |
| 7 | .00204 | One. | .01949 | .99981 | .03693 | .99932 | .05437 | .99852 | .07179 | .99742 | 53 |
| 8 | .00233 | One. | .01978 | .99980 | .03723 | .99931 | .05466 | .99851 | .07208 | .99740 | 52 |
| 9 | .00262 | One. | .02007 | .99980 | .03752 | .99930 | .05495 | .99849 | .07237 | .99738 | 51 |
| 10 | .00291 | One. | .02036 | .99979 | .03781 | .99929 | .05524 | .99847 | .07266 | .99736 | 50 |
| 11 | .00320 | .99999 | .02065 | .99979 | .03810 | .99927 | .05553 | .99846 | .07295 | .99734 | 49 |
| 12 | .00349 | .99999 | .02094 | .99978 | .03839 | .99926 | .05582 | .99844 | .07324 | .99731 | 48 |
| 13 | .00378 | .99999 | .02123 | .99977 | .03868 | .99925 | .05611 | .99843 | .07353 | .99729 | 47 |
| 14 | .00407 | .99999 | .02152 | .99977 | .03897 | .99924 | .05640 | .99841 | .07382 | .99727 | 46 |
| 15 | .00436 | .99999 | .02181 | .99976 | .03926 | .99923 | .05669 | .99839 | .07411 | .99725 | 45 |
| 16 | .00465 | .99999 | .02211 | .99976 | .03955 | .99922 | .05698 | .99838 | .07440 | .99723 | 44 |
| 17 | .00495 | .99999 | .02240 | .99975 | .03984 | .99921 | .05727 | .99838 | .07469 | .99721 | 43 |
| 18 | .00524 | .99999 | .02269 | .99974 | .04013 | .99919 | .05756 | .99834 | .07498 | .99719 | 42 |
| 19 | .00553 | .99998 | .02298 | .99974 | .04042 | .99918 | .05785 | .99833 | .07527 | .99716 | 41 |
| 20 | .00582 | .99998 | .02327 | .99973 | .04071 | .99917 | .05814 | .99831 | .07556 | .99714 | 40 |
| 21 | .00611 | .99998 | .02356 | .99972 | .04100 | .99916 | .05844 | .99829 | .07585 | .99712 | 39 |
| 22 | .00640 | .99998 | .02385 | .99972 | .04129 | .99915 | .05873 | .99827 | .07614 | .99710 | 38 |
| 23 | .00669 | .99998 | .02414 | .99971 | .04159 | .99913 | .05902 | .99826 | .07643 | .99708 | 37 |
| 24 | .00688 | .99998 | .02443 | .99970 | .04188 | .99912 | .05931 | .99824 | .07672 | .99705 | 36 |
| 25 | .00727 | .99997 | .02472 | .99969 | .04217 | .99911 | .05960 | .99822 | .07701 | .99703 | 35 |
| 26 | .00756 | .99997 | .02501 | .99969 | .04246 | .99910 | .05989 | .99821 | .07730 | .99701 | 34 |
| 27 | .00785 | .99997 | .02530 | .99968 | .04275 | .99909 | .06018 | .99819 | .07759 | .99699 | 33 |
| 28 | .00814 | .99997 | .02559 | .99967 | .04304 | .99907 | .06047 | .99817 | .07788 | .99696 | 32 |
| 29 | .00844 | .99996 | .02589 | .99966 | .04333 | .99906 | .06076 | .99815 | .07817 | .99694 | 31 |
| 30 | .00873 | .99996 | .02618 | .99966 | .04362 | .99905 | .06105 | .99813 | .07846 | .99693 | 30 |
| 31 | .00902 | .99996 | .02647 | .99965 | .04391 | .99904 | .06134 | .99812 | .07875 | .99689 | 29 |
| 32 | .00931 | .99996 | .02676 | .99964 | .04420 | .99902 | .06163 | .99810 | .07904 | .99687 | 28 |
| 33 | .00960 | .99995 | .02705 | .99963 | .04449 | .99901 | .06192 | .99808 | .07933 | .99685 | 27 |
| 34 | .00989 | .99995 | .02734 | .99963 | .04478 | .99900 | .06221 | .99806 | .07962 | .99683 | 26 |
| 35 | .01018 | .99995 | .02763 | .99962 | .04507 | .99898 | .06250 | .99804 | .07991 | .99680 | 25 |
| 36 | .01047 | .99995 | .02792 | .99961 | .04536 | .99897 | .06279 | .99803 | .08020 | .99678 | 24 |
| 37 | .01076 | .99994 | .02821 | .99960 | .04565 | .99896 | .06308 | .99801 | .08049 | .99676 | 23 |
| 38 | .01105 | .99994 | .02850 | .99959 | .04594 | .99894 | .06337 | .99799 | .08078 | .99673 | 22 |
| 39 | .01134 | .99994 | .02879 | .99959 | .04623 | .99893 | .06366 | .99797 | .08107 | .99671 | 21 |
| 40 | .01164 | .99993 | .02908 | .99958 | .04653 | .99892 | .06395 | .99795 | .08136 | .99668 | 20 |
| 41 | .01193 | .99993 | .02938 | .99957 | .04682 | .99890 | .06424 | .99793 | .08165 | .99666 | 19 |
| 42 | .01222 | .99993 | .02967 | .99956 | .04711 | .99889 | .06453 | .99792 | .08194 | .99664 | 18 |
| 43 | .01251 | .99993 | .02996 | .99955 | .04740 | .99888 | .06482 | .99790 | .08223 | .99661 | 17 |
| 44 | .01280 | .99992 | .03025 | .99954 | .04769 | .99886 | .06511 | .99788 | .08252 | .99659 | 16 |
| 45 | .01309 | .99991 | .03054 | .99953 | .04798 | .99885 | .06540 | .99786 | .08281 | .99657 | 15 |
| 46 | .01338 | .99991 | .03083 | .99952 | .04827 | .99883 | .06569 | .99784 | .08310 | .99654 | 14 |
| 47 | .01367 | .99991 | .03112 | .99952 | .04856 | .99882 | .06598 | .99782 | .08339 | .99652 | 13 |
| 48 | .01396 | .99990 | .03141 | .99951 | .04885 | .99881 | .06627 | .99780 | .08368 | .99649 | 12 |
| 49 | .01425 | .99990 | .03170 | .99950 | .04914 | .99879 | .06656 | .99778 | .08397 | .99647 | 11 |
| 50 | .01454 | .99989 | .03199 | .99949 | .04943 | .99878 | .06685 | .99776 | .08426 | .99644 | 10 |
| 51 | .01483 | .99989 | .03228 | .99948 | .04972 | .99876 | .06714 | .99774 | .08455 | .99642 | 9 |
| 52 | .01513 | .99989 | .03257 | .99947 | .05001 | .99875 | .06743 | .99772 | .08484 | .99639 | 8 |
| 53 | .01542 | .99988 | .03286 | .99946 | .05030 | .99873 | .06773 | .99770 | .08513 | .99637 | 7 |
| 54 | .01571 | .99988 | .03316 | .99945 | .05059 | .99872 | .06802 | .99768 | .08542 | .99635 | 6 |
| 55 | .01600 | .99987 | .03345 | .99944 | .05088 | .99870 | .06831 | .99766 | .08571 | .99633 | 5 |
| 56 | .01629 | .99987 | .03374 | .99943 | .05117 | .99869 | .06860 | .99764 | .08600 | .99630 | 4 |
| 57 | .01658 | .99986 | .03403 | .99942 | .05146 | .99867 | .06889 | .99762 | .08629 | .99627 | 3 |
| 58 | .01687 | .99986 | .03432 | .99941 | .05175 | .99866 | .06918 | .99760 | .08658 | .99625 | 2 |
| 59 | .01716 | .99985 | .03461 | .99940 | .05205 | .99864 | .06947 | .99758 | .08687 | .99622 | 1 |
| 60 | .01745 | .99985 | .03490 | .99939 | .05234 | .99863 | .06976 | .99756 | .08716 | .99619 | 0 |
| | Cosin | Sine | |

| | 6° | | 6° | | 7° | | 8° | | 9° | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| | Sine | Cosin | |
| 0 | .08716 | .99619 | .10453 | .99452 | .12187 | .99255 | .13917 | .99027 | .15643 | .98769 | 60 |
| 1 | .08745 | .99617 | .10482 | .99449 | .12216 | .99251 | .13946 | .99023 | .15672 | .98764 | 59 |
| 2 | .08774 | .99614 | .10511 | .99446 | .12245 | .99248 | .13975 | .99019 | .15701 | .98760 | 58 |
| 3 | .08803 | .99612 | .10540 | .99442 | .12274 | .99244 | .14004 | .99015 | .15730 | .98755 | 57 |
| 4 | .08831 | .99609 | .10569 | .99440 | .12302 | .99240 | .14033 | .99011 | .15758 | .98751 | 56 |
| 5 | .08860 | .99607 | .10597 | .99437 | .12331 | .99237 | .14061 | .99006 | .15787 | .98746 | 55 |
| 6 | .08889 | .99604 | .10626 | .99434 | .12360 | .99233 | .14090 | .99002 | .15816 | .98741 | 54 |
| 7 | .08918 | .99603 | .10655 | .99431 | .12389 | .99230 | .14119 | .98998 | .15845 | .98737 | 53 |
| 8 | .08947 | .99599 | .10684 | .99428 | .12418 | .99226 | .14148 | .98994 | .15873 | .98739 | 52 |
| 9 | .08976 | .99596 | .10713 | .99424 | .12447 | .99222 | .14177 | .98990 | .15902 | .98725 | 51 |
| 10 | .09005 | .99594 | .10742 | .99421 | .12476 | .99219 | .14205 | .98986 | .15931 | .98723 | 50 |
| 11 | .09034 | .99591 | .10771 | .99418 | .12504 | .99215 | .14234 | .98982 | .15959 | .98718 | 49 |
| 12 | .09063 | .99588 | .10800 | .99415 | .12533 | .99211 | .14263 | .98978 | .15988 | .98714 | 48 |
| 13 | .09092 | .99586 | .10829 | .99412 | .12562 | .99208 | .14292 | .98973 | .16017 | .98709 | 47 |
| 14 | .09121 | .99583 | .10858 | .99409 | .12591 | .99204 | .14320 | .98969 | .16046 | .98704 | 46 |
| 15 | .09150 | .99580 | .10887 | .99406 | .12620 | .99200 | .14349 | .98965 | .16074 | .98700 | 45 |
| 16 | .09179 | .99578 | .10916 | .99402 | .12649 | .99197 | .14378 | .98961 | .16103 | .98695 | 44 |
| 17 | .09208 | .99575 | .10945 | .99399 | .12678 | .99193 | .14407 | .98957 | .16132 | .98690 | 43 |
| 18 | .09237 | .99572 | .10973 | .99396 | .12706 | .99189 | .14436 | .98953 | .16160 | .98686 | 42 |
| 19 | .09266 | .99570 | .11002 | .99393 | .12735 | .99186 | .14464 | .98948 | .16189 | .98681 | 41 |
| 20 | .09295 | .99567 | .11031 | .99390 | .12764 | .99182 | .14493 | .98944 | .16218 | .98676 | 40 |
| 21 | .09324 | .99564 | .11060 | .99386 | .12793 | .99178 | .14522 | .98940 | .16246 | .98671 | 39 |
| 22 | .09353 | .99562 | .11089 | .99383 | .12822 | .99175 | .14551 | .98936 | .16275 | .98667 | 38 |
| 23 | .09382 | .99559 | .11118 | .99380 | .12851 | .99171 | .14580 | .98931 | .16304 | .98662 | 37 |
| 24 | .09411 | .99556 | .11147 | .99377 | .12880 | .99167 | .14608 | .98927 | .16333 | .98657 | 36 |
| 25 | .09440 | .99553 | .11176 | .99374 | .12908 | .99163 | .14637 | .98923 | .16361 | .98652 | 35 |
| 26 | .09469 | .99551 | .11205 | .99370 | .12937 | .99160 | .14666 | .98919 | .16389 | .98648 | 34 |
| 27 | .09498 | .99548 | .11234 | .99367 | .12966 | .99156 | .14695 | .98914 | .16419 | .98643 | 33 |
| 28 | .09527 | .99545 | .11263 | .99364 | .12995 | .99152 | .14723 | .98910 | .16447 | .98638 | 32 |
| 29 | .09556 | .99542 | .11291 | .99360 | .13024 | .99148 | .14752 | .98906 | .16476 | .98633 | 31 |
| 30 | .09585 | .99540 | .11320 | .99357 | .13053 | .99144 | .14781 | .98902 | .16505 | .98629 | 30 |
| 31 | .09614 | .99537 | .11349 | .99354 | .13081 | .99141 | .14810 | .98897 | .16533 | .98624 | 29 |
| 32 | .09643 | .99534 | .11378 | .99351 | .13110 | .99137 | .14838 | .98893 | .16562 | .98619 | 28 |
| 33 | .09671 | .99531 | .11407 | .99347 | .13139 | .99133 | .14867 | .98889 | .16591 | .98614 | 27 |
| 34 | .09700 | .99528 | .11436 | .99344 | .13163 | .99129 | .14896 | .98884 | .16620 | .98609 | 26 |
| 35 | .09729 | .99526 | .11465 | .99341 | .13197 | .99125 | .14925 | .98880 | .16648 | .98604 | 25 |
| 36 | .09758 | .99523 | .11494 | .99337 | .13226 | .99122 | .14954 | .98876 | .16677 | .98600 | 24 |
| 37 | .09787 | .99520 | .11523 | .99334 | .13254 | .99118 | .14982 | .98871 | .16706 | .98595 | 23 |
| 38 | .09816 | .99517 | .11552 | .99331 | .13283 | .99114 | .15011 | .98867 | .16734 | .98590 | 22 |
| 39 | .09845 | .99514 | .11580 | .99327 | .13312 | .99110 | .15040 | .98863 | .16763 | .98585 | 21 |
| 40 | .09874 | .99511 | .11609 | .99324 | .13341 | .99106 | .15069 | .98858 | .16792 | .98580 | 20 |
| 41 | .09903 | .99508 | .11638 | .99320 | .13370 | .99102 | .15097 | .98854 | .16820 | .98575 | 19 |
| 42 | .09932 | .99506 | .11667 | .99317 | .13399 | .99098 | .15126 | .98849 | .16849 | .98570 | 18 |
| 43 | .09961 | .99503 | .11696 | .99314 | .13427 | .99094 | .15155 | .98845 | .16878 | .98565 | 17 |
| 44 | .09990 | .99500 | .11725 | .99310 | .13456 | .99091 | .15184 | .98841 | .16906 | .98561 | 16 |
| 45 | .10019 | .99497 | .11754 | .99307 | .13485 | .99087 | .15212 | .98836 | .16935 | .98556 | 15 |
| 46 | .10048 | .99494 | .11783 | .99303 | .13514 | .99083 | .15241 | .98832 | .16964 | .98551 | 14 |
| 47 | .10077 | .99491 | .11812 | .99300 | .13543 | .99079 | .15270 | .98827 | .16992 | .98546 | 13 |
| 48 | .10106 | .99488 | .11840 | .99297 | .13572 | .99075 | .15299 | .98823 | .17021 | .98541 | 12 |
| 49 | .10135 | .99485 | .11869 | .99293 | .13600 | .99071 | .15327 | .98818 | .17050 | .98536 | 11 |
| 50 | .10164 | .99482 | .11898 | .99290 | .13629 | .99067 | .15356 | .98814 | .17078 | .98531 | 10 |
| 51 | .10193 | .99479 | .11927 | .99286 | .13658 | .99063 | .15385 | .98809 | .17107 | .98526 | 9 |
| 52 | .10221 | .99476 | .11956 | .99283 | .13687 | .99059 | .15414 | .98805 | .17136 | .98521 | 8 |
| 53 | .10250 | .99473 | .11985 | .99279 | .13716 | .99055 | .15442 | .98800 | .17164 | .98516 | 7 |
| 54 | .10279 | .99470 | .12014 | .99276 | .13744 | .99051 | .15471 | .98796 | .17193 | .98511 | 6 |
| 55 | .10308 | .99467 | .12043 | .99272 | .13773 | .99047 | .15500 | .98791 | .17222 | .98506 | 5 |
| 56 | .10337 | .99464 | .12071 | .99269 | .13802 | .99043 | .15529 | .98787 | .17250 | .98501 | 4 |
| 57 | .10366 | .99461 | .12100 | .99265 | .13831 | .99039 | .15557 | .98783 | .17279 | .98496 | 3 |
| 58 | .10395 | .99458 | .12129 | .99262 | .13860 | .99035 | .15586 | .98778 | .17308 | .98491 | 2 |
| 59 | .10424 | .99455 | .12158 | .99258 | .13889 | .99031 | .15615 | .98773 | .17336 | .98486 | 1 |
| 60 | .10453 | .99452 | .12187 | .99255 | .13917 | .99027 | .15643 | .98769 | .17365 | .98481 | 0 |

| | 10° | | 11° | | 12° | | 13° | | 14° | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| | Sine | Cosin | |
| 0 | .17365 | .98481 | .19081 | .98163 | .20791 | .97815 | .22405 | .97437 | .24192 | .97030 | 60 |
| 1 | .17393 | .98476 | .19109 | .98157 | .20820 | .97809 | .22523 | .97430 | .24230 | .97023 | 59 |
| 2 | .17422 | .98471 | .19138 | .98152 | .20848 | .97803 | .22552 | .97424 | .24249 | .97015 | 58 |
| 3 | .17451 | .98466 | .19167 | .98146 | .20877 | .97797 | .22580 | .97417 | .24277 | .97008 | 57 |
| 4 | .17479 | .98461 | .19195 | .98140 | .20905 | .97791 | .22608 | .97411 | .24305 | .97001 | 56 |
| 5 | .17508 | .98455 | .19224 | .98135 | .20933 | .97784 | .22637 | .97404 | .24333 | .96994 | 55 |
| 6 | .17537 | .98450 | .19253 | .98129 | .20962 | .97778 | .22665 | .97398 | .24362 | .96987 | 54 |
| 7 | .17565 | .98445 | .19281 | .98124 | .20990 | .97772 | .22693 | .97391 | .24390 | .96980 | 53 |
| 8 | .17594 | .98440 | .19309 | .98118 | .21019 | .97766 | .22722 | .97384 | .24418 | .96973 | 52 |
| 9 | .17623 | .98435 | .19338 | .98112 | .21047 | .97760 | .22750 | .97378 | .24446 | .96966 | 51 |
| 10 | .17651 | .98430 | .19366 | .98107 | .21076 | .97754 | .22778 | .97371 | .24474 | .96959 | 50 |
| 11 | .17680 | .98425 | .19395 | .98101 | .21104 | .97748 | .22807 | .97365 | .24503 | .96952 | 49 |
| 12 | .17708 | .98420 | .19423 | .98096 | .21132 | .97742 | .22835 | .97358 | .24531 | .96945 | 48 |
| 13 | .17737 | .98414 | .19452 | .98090 | .21161 | .97735 | .22863 | .97351 | .24559 | .96937 | 47 |
| 14 | .17766 | .98409 | .19481 | .98084 | .21189 | .97729 | .22892 | .97345 | .24587 | .96930 | 46 |
| 15 | .17794 | .98404 | .19509 | .98079 | .21218 | .97723 | .22920 | .97338 | .24615 | .96923 | 45 |
| 16 | .17823 | .98399 | .19538 | .98073 | .21246 | .97717 | .22948 | .97331 | .24644 | .96916 | 44 |
| 17 | .17852 | .98394 | .19566 | .98067 | .21275 | .97711 | .22977 | .97324 | .24672 | .96909 | 43 |
| 18 | .17880 | .98389 | .19595 | .98061 | .21303 | .97705 | .23005 | .97318 | .24700 | .96902 | 42 |
| 19 | .17909 | .98383 | .19623 | .98056 | .21331 | .97698 | .23033 | .97311 | .24728 | .96894 | 41 |
| 20 | .17937 | .98378 | .19652 | .98050 | .21360 | .97692 | .23062 | .97304 | .24756 | .96887 | 40 |
| 21 | .17966 | .98373 | .19680 | .98044 | .21388 | .97686 | .23090 | .97298 | .24784 | .96880 | 39 |
| 22 | .17995 | .98368 | .19709 | .98039 | .21417 | .97680 | .23118 | .97291 | .24813 | .96873 | 38 |
| 23 | .18023 | .98363 | .19737 | .98033 | .21445 | .97673 | .23146 | .97284 | .24841 | .96866 | 37 |
| 24 | .18052 | .98357 | .19766 | .98027 | .21474 | .97667 | .23175 | .97278 | .24869 | .96858 | 36 |
| 25 | .18081 | .98352 | .19794 | .98021 | .21502 | .97661 | .23203 | .97271 | .24897 | .96851 | 35 |
| 26 | .18109 | .98347 | .19823 | .98016 | .21530 | .97655 | .23231 | .97264 | .24925 | .96844 | 34 |
| 27 | .18138 | .98341 | .19851 | .98010 | .21559 | .97649 | .23259 | .97257 | .24954 | .96837 | 33 |
| 28 | .18166 | .98336 | .19880 | .98004 | .21587 | .97643 | .23283 | .97251 | .24982 | .96829 | 32 |
| 29 | .18195 | .98331 | .19908 | .97998 | .21616 | .97636 | .23310 | .97244 | .25010 | .96822 | 31 |
| 30 | .18224 | .98325 | .19937 | .97992 | .21644 | .97630 | .23345 | .97237 | .25038 | .96815 | 30 |
| 31 | .18252 | .98320 | .19965 | .97987 | .21672 | .97623 | .23373 | .97230 | .25066 | .96807 | 29 |
| 32 | .18281 | .98315 | .19994 | .97981 | .21701 | .97617 | .23401 | .97223 | .25094 | .96800 | 28 |
| 33 | .18309 | .98310 | .20022 | .97975 | .21730 | .97611 | .23429 | .97217 | .25122 | .96793 | 27 |
| 34 | .18338 | .98304 | .20051 | .97969 | .21758 | .97604 | .23443 | .97210 | .25151 | .96786 | 26 |
| 35 | .18367 | .98299 | .20079 | .97963 | .21783 | .97598 | .23483 | .97203 | .25179 | .96778 | 25 |
| 36 | .18395 | .98294 | .20108 | .97958 | .21814 | .97592 | .23514 | .97196 | .25207 | .96771 | 24 |
| 37 | .18424 | .98288 | .20136 | .97952 | .21843 | .97585 | .23543 | .97188 | .25235 | .96764 | 23 |
| 38 | .18453 | .98283 | .20165 | .97946 | .21873 | .97579 | .23571 | .97181 | .25263 | .96756 | 22 |
| 39 | .18481 | .98277 | .20193 | .97940 | .21909 | .97573 | .23599 | .97176 | .25291 | .96749 | 21 |
| 40 | .18509 | .98272 | .20222 | .97934 | .21938 | .97566 | .23627 | .97169 | .25320 | .96742 | 20 |
| 41 | .18538 | .98267 | .20250 | .97928 | .21956 | .97560 | .23656 | .97163 | .25348 | .96734 | 19 |
| 42 | .18567 | .98261 | .20279 | .97922 | .21985 | .97553 | .23684 | .97155 | .25376 | .96727 | 18 |
| 43 | .18595 | .98256 | .20307 | .97916 | .22013 | .97547 | .23712 | .97148 | .25404 | .96717 | 17 |
| 44 | .18624 | .98250 | .20336 | .97910 | .22041 | .97541 | .23740 | .97141 | .25432 | .96712 | 16 |
| 45 | .18652 | .98245 | .20364 | .97905 | .22070 | .97534 | .23769 | .97134 | .25460 | .96705 | 15 |
| 46 | .18681 | .98240 | .20393 | .97899 | .22098 | .97528 | .23797 | .97127 | .25488 | .96697 | 14 |
| 47 | .18710 | .98234 | .20421 | .97893 | .22126 | .97521 | .23825 | .97120 | .25516 | .96690 | 13 |
| 48 | .18738 | .98229 | .20450 | .97887 | .22155 | .97515 | .23853 | .97113 | .25545 | .96682 | 12 |
| 49 | .18767 | .98223 | .20478 | .97881 | .22183 | .97508 | .23882 | .97106 | .25573 | .96675 | 11 |
| 50 | .18795 | .98218 | .20507 | .97875 | .22212 | .97502 | .23910 | .97100 | .25601 | .96667 | 10 |
| 51 | .18824 | .98212 | .20535 | .97869 | .22240 | .97496 | .23938 | .97093 | .25620 | .96660 | 9 |
| 52 | .18852 | .98207 | .20563 | .97863 | .22268 | .97489 | .23966 | .97086 | .25657 | .96653 | 8 |
| 53 | .18881 | .98201 | .20592 | .97857 | .22297 | .97483 | .23995 | .97079 | .25685 | .96645 | 7 |
| 54 | .18910 | .98196 | .20620 | .97851 | .22325 | .97476 | .24023 | .97072 | .25713 | .96638 | 6 |
| 55 | .18938 | .98190 | .20649 | .97845 | .22353 | .97470 | .24051 | .97065 | .25741 | .96630 | 5 |
| 56 | .18967 | .98185 | .20677 | .97839 | .22380 | .97463 | .24079 | .97058 | .25769 | .96623 | 4 |
| 57 | .18995 | .98179 | .20706 | .97833 | .22410 | .97457 | .24108 | .97051 | .25798 | .96615 | 3 |
| 58 | .19024 | .98174 | .20734 | .97827 | .22438 | .97450 | .24136 | .97044 | .25826 | .96608 | 2 |
| 59 | .19052 | .98168 | .20763 | .97821 | .22467 | .97444 | .24164 | .97037 | .25854 | .96600 | 1 |
| 60 | .19081 | .98163 | .20791 | .97815 | .22495 | .97437 | .24193 | .97030 | .25882 | .96593 | 0 |
| | Cosin | Sine | |

| | 15° | | 16° | | 17° | | 18° | | 19° | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| | Sine | Cosin | |
| 0 | .25882 | .96593 | .27564 | .96126 | .29237 | .95630 | .30902 | .95106 | .32557 | .94552 | 60 |
| 1 | .25910 | .96555 | .27592 | .96118 | .29265 | .95623 | .30929 | .95097 | .32584 | .94542 | 59 |
| 2 | .25938 | .96578 | .27620 | .96110 | .29293 | .95613 | .30957 | .95088 | .32612 | .94533 | 58 |
| 3 | .25966 | .96570 | .27648 | .96102 | .29321 | .95605 | .30985 | .95079 | .32639 | .94523 | 57 |
| 4 | .25994 | .96562 | .27676 | .96094 | .29348 | .95596 | .31012 | .95070 | .32667 | .94514 | 56 |
| 5 | .26022 | .96555 | .27704 | .96086 | .29376 | .95588 | .31040 | .95061 | .32694 | .94505 | 55 |
| 6 | .26050 | .96547 | .27731 | .96078 | .29404 | .95579 | .31068 | .95052 | .32722 | .94495 | 54 |
| 7 | .26079 | .96540 | .27759 | .96070 | .29432 | .95571 | .31095 | .95043 | .32749 | .94485 | 53 |
| 8 | .26107 | .96532 | .27787 | .96062 | .29460 | .95562 | .31123 | .95033 | .32777 | .94476 | 52 |
| 9 | .26135 | .96524 | .27815 | .96054 | .29487 | .95554 | .31151 | .95024 | .32804 | .94466 | 51 |
| 10 | .26163 | .96517 | .27843 | .96046 | .29515 | .95545 | .31178 | .95015 | .32832 | .94457 | 50 |
| 11 | .26191 | .96509 | .27871 | .96037 | .29543 | .95536 | .31206 | .95006 | .32859 | .94447 | 49 |
| 12 | .26219 | .96502 | .27909 | .96029 | .29571 | .95528 | .31233 | .94997 | .32887 | .94438 | 48 |
| 13 | .26247 | .96494 | .27927 | .96021 | .29599 | .95519 | .31261 | .94988 | .32914 | .94428 | 47 |
| 14 | .26275 | .96486 | .27955 | .96013 | .29626 | .95511 | .31289 | .94979 | .32942 | .94418 | 46 |
| 15 | .26303 | .96479 | .27983 | .96005 | .29654 | .95503 | .31316 | .94970 | .32969 | .94409 | 45 |
| 16 | .26331 | .96471 | .28011 | .95997 | .29682 | .95493 | .31344 | .94961 | .32997 | .94399 | 44 |
| 17 | .26359 | .96463 | .28039 | .95989 | .29710 | .95485 | .31372 | .94952 | .33024 | .94390 | 43 |
| 18 | .26387 | .96456 | .28067 | .95981 | .29737 | .95476 | .31399 | .94943 | .33051 | .94380 | 42 |
| 19 | .26415 | .96448 | .28095 | .95972 | .29765 | .95467 | .31427 | .94933 | .33079 | .94370 | 41 |
| 20 | .26443 | .96440 | .28123 | .95964 | .29793 | .95459 | .31454 | .94924 | .33106 | .94361 | 40 |
| 21 | .26471 | .96433 | .28150 | .95956 | .29821 | .95450 | .31482 | .94915 | .33134 | .94351 | 39 |
| 22 | .26500 | .96425 | .28178 | .95948 | .29849 | .95441 | .31510 | .94906 | .33161 | .94342 | 38 |
| 23 | .26528 | .96417 | .28206 | .95940 | .29876 | .95433 | .31537 | .94897 | .33189 | .94332 | 37 |
| 24 | .26556 | .96410 | .28234 | .95931 | .29904 | .95424 | .31565 | .94888 | .33216 | .94322 | 36 |
| 25 | .26584 | .96402 | .28262 | .95923 | .29932 | .95415 | .31593 | .94878 | .33244 | .94313 | 35 |
| 26 | .26612 | .96394 | .28290 | .95915 | .29960 | .95407 | .31620 | .94869 | .33271 | .94303 | 34 |
| 27 | .26640 | .96386 | .28318 | .95907 | .29987 | .95398 | .31648 | .94860 | .33298 | .94293 | 33 |
| 28 | .26668 | .96379 | .28346 | .95898 | .30015 | .95389 | .31675 | .94851 | .33326 | .94284 | 32 |
| 29 | .26696 | .96371 | .28374 | .95890 | .30043 | .95380 | .31703 | .94842 | .33353 | .94274 | 31 |
| 30 | .26724 | .96363 | .28402 | .95882 | .30071 | .95372 | .31730 | .94832 | .33381 | .94264 | 30 |
| 31 | .26752 | .96355 | .28429 | .95874 | .30098 | .95363 | .31758 | .94823 | .33408 | .94254 | 29 |
| 32 | .26780 | .96347 | .28457 | .95865 | .30126 | .95354 | .31786 | .94814 | .33436 | .94245 | 28 |
| 33 | .26808 | .96340 | .28485 | .95857 | .30154 | .95345 | .31813 | .94805 | .33463 | .94235 | 27 |
| 34 | .26836 | .96332 | .28513 | .95849 | .30182 | .95337 | .31841 | .94795 | .33490 | .94225 | 26 |
| 35 | .26864 | .96324 | .28541 | .95841 | .30209 | .95328 | .31868 | .94786 | .33518 | .94215 | 25 |
| 36 | .26892 | .96316 | .28569 | .95832 | .30237 | .95319 | .31896 | .94777 | .33545 | .94205 | 24 |
| 37 | .26920 | .96308 | .28597 | .95824 | .30265 | .95310 | .31923 | .94768 | .33573 | .94196 | 23 |
| 38 | .26948 | .96301 | .28623 | .95816 | .30292 | .95301 | .31951 | .94758 | .33600 | .94186 | 22 |
| 39 | .26967 | .96293 | .28652 | .95807 | .30320 | .95293 | .31979 | .94749 | .33627 | .94176 | 21 |
| 40 | .27004 | .96285 | .28680 | .95799 | .30348 | .95284 | .32006 | .94740 | .33655 | .94167 | 20 |
| 41 | .27032 | .96277 | .28708 | .95791 | .30376 | .95275 | .32034 | .94730 | .33682 | .94157 | 19 |
| 42 | .27060 | .96269 | .28736 | .95783 | .30403 | .95266 | .32061 | .94721 | .33710 | .94147 | 18 |
| 43 | .27088 | .96261 | .28764 | .95774 | .30431 | .95257 | .32089 | .94712 | .33737 | .94137 | 17 |
| 44 | .27116 | .96253 | .28792 | .95766 | .30459 | .95248 | .32116 | .94703 | .33764 | .94127 | 16 |
| 45 | .27144 | .96246 | .28820 | .95757 | .30486 | .95240 | .32144 | .94693 | .33793 | .94118 | 15 |
| 46 | .27172 | .96238 | .28847 | .95749 | .30514 | .95231 | .32171 | .94684 | .33819 | .94108 | 14 |
| 47 | .27200 | .96230 | .28875 | .95740 | .30543 | .95222 | .32199 | .94674 | .33846 | .94098 | 13 |
| 48 | .27228 | .96222 | .28903 | .95732 | .30570 | .95213 | .32227 | .94665 | .33874 | .94088 | 12 |
| 49 | .27256 | .96214 | .28931 | .95724 | .30597 | .95204 | .32254 | .94656 | .33901 | .94078 | 11 |
| 50 | .27284 | .96206 | .28959 | .95715 | .30625 | .95195 | .32282 | .94646 | .33929 | .94068 | 10 |
| 51 | .27312 | .96198 | .28987 | .95707 | .30653 | .95186 | .32309 | .94637 | .33956 | .94058 | 9 |
| 52 | .27340 | .96190 | .29015 | .95698 | .30680 | .95177 | .32337 | .94627 | .33985 | .94049 | 8 |
| 53 | .27368 | .96182 | .29042 | .95690 | .30708 | .95168 | .32364 | .94618 | .34011 | .94039 | 7 |
| 54 | .27396 | .96174 | .29070 | .95681 | .30736 | .95159 | .32392 | .94609 | .34035 | .94029 | 6 |
| 55 | .27424 | .96166 | .29098 | .95673 | .30763 | .95150 | .32419 | .94599 | .34065 | .94019 | 5 |
| 56 | .27452 | .96158 | .29126 | .95664 | .30791 | .95142 | .32447 | .94590 | .34093 | .94009 | 4 |
| 57 | .27480 | .96150 | .29154 | .95656 | .30819 | .95133 | .32474 | .94580 | .34120 | .93999 | 3 |
| 58 | .27508 | .96142 | .29182 | .95647 | .30846 | .95124 | .32502 | .94571 | .34147 | .93989 | 2 |
| 59 | .27536 | .96134 | .29209 | .95639 | .30874 | .95115 | .32529 | .94561 | .34175 | .93979 | 1 |
| 60 | .27564 | .96126 | .29237 | .95630 | .30902 | .95106 | .32557 | .94552 | .34202 | .93969 | 0 |
| | Cosin | Sine | |
| | 74° | 73° | 72° | 71° | 70° | | | | | | |

| | 30° | | 31° | | 32° | | 33° | | 34° | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| | Sine | Cosin | |
| 0 | .50000 | .86603 | .51504 | .85717 | .52992 | .84805 | .54464 | .83867 | .55919 | .82904 | 60 |
| 1 | .50025 | .86588 | .51529 | .85703 | .53017 | .84789 | .54488 | .83851 | .55943 | .82887 | 59 |
| 2 | .50050 | .86573 | .51554 | .85687 | .53041 | .84774 | .54513 | .83835 | .55968 | .82871 | 58 |
| 3 | .50076 | .86559 | .51579 | .85673 | .53066 | .84759 | .54537 | .83819 | .55992 | .82855 | 57 |
| 4 | .50101 | .86544 | .51604 | .85657 | .53091 | .84743 | .54561 | .83804 | .56016 | .82839 | 56 |
| 5 | .50126 | .86530 | .51628 | .85643 | .53115 | .84728 | .54586 | .83788 | .56040 | .82822 | 55 |
| 6 | .50151 | .86515 | .51653 | .85627 | .53140 | .84712 | .54610 | .83773 | .56064 | .82806 | 54 |
| 7 | .50176 | .86501 | .51678 | .85612 | .53164 | .84697 | .54635 | .83756 | .56088 | .82790 | 53 |
| 8 | .50201 | .86486 | .51703 | .85597 | .53189 | .84681 | .54659 | .83740 | .56112 | .82773 | 52 |
| 9 | .50227 | .86471 | .51728 | .85582 | .53214 | .84666 | .54683 | .83724 | .56136 | .82757 | 51 |
| 10 | .50252 | .86457 | .51753 | .85567 | .53238 | .84650 | .54708 | .83708 | .56160 | .82741 | 50 |
| 11 | .50276 | .86442 | .51778 | .85551 | .53263 | .84635 | .54732 | .83692 | .56184 | .82724 | 49 |
| 12 | .50302 | .86427 | .51803 | .85536 | .53288 | .84619 | .54756 | .83676 | .56208 | .82708 | 48 |
| 13 | .50327 | .86413 | .51828 | .85521 | .53312 | .84604 | .54781 | .83660 | .56232 | .82692 | 47 |
| 14 | .50352 | .86398 | .51852 | .85506 | .53337 | .84588 | .54805 | .83645 | .56256 | .82675 | 46 |
| 15 | .50377 | .86384 | .51877 | .85491 | .53361 | .84573 | .54829 | .83629 | .56280 | .82659 | 45 |
| 16 | .50402 | .86369 | .51902 | .85476 | .53386 | .84557 | .54854 | .83613 | .56305 | .82643 | 44 |
| 17 | .50428 | .86354 | .51927 | .85461 | .53411 | .84542 | .54878 | .83597 | .56329 | .82626 | 43 |
| 18 | .50453 | .86340 | .51952 | .85446 | .53435 | .84526 | .54902 | .83581 | .56353 | .82610 | 42 |
| 19 | .50478 | .86325 | .51977 | .85431 | .53460 | .84511 | .54927 | .83565 | .56377 | .82593 | 41 |
| 20 | .50503 | .86310 | .52002 | .85416 | .53484 | .84495 | .54951 | .83549 | .56401 | .82577 | 40 |
| 21 | .50528 | .86295 | .52026 | .85401 | .53509 | .84480 | .54975 | .83533 | .56425 | .82561 | 39 |
| 22 | .50553 | .86281 | .52051 | .85385 | .53534 | .84464 | .55000 | .83517 | .56449 | .82544 | 38 |
| 23 | .50578 | .86266 | .52076 | .85370 | .53558 | .84448 | .55024 | .83501 | .56473 | .82528 | 37 |
| 24 | .50603 | .86251 | .52101 | .85355 | .53583 | .84433 | .55048 | .83485 | .56497 | .82511 | 36 |
| 25 | .50628 | .86237 | .52126 | .85340 | .53607 | .84417 | .55072 | .83469 | .56521 | .82495 | 35 |
| 26 | .50654 | .86222 | .52151 | .85325 | .53632 | .84402 | .55097 | .83453 | .56545 | .82478 | 34 |
| 27 | .50679 | .86207 | .52175 | .85310 | .53656 | .84386 | .55121 | .83437 | .56569 | .82463 | 33 |
| 28 | .50704 | .86192 | .52200 | .85294 | .53681 | .84370 | .55145 | .83421 | .56593 | .82446 | 32 |
| 29 | .50729 | .86178 | .52225 | .85279 | .53705 | .84355 | .55169 | .83405 | .56617 | .82420 | 31 |
| 30 | .50754 | .86163 | .52250 | .85264 | .53730 | .84339 | .55194 | .83389 | .56641 | .82413 | 30 |
| 31 | .50779 | .86148 | .52275 | .85249 | .53754 | .84324 | .55218 | .83373 | .56665 | .82396 | 29 |
| 32 | .50804 | .86133 | .52300 | .85234 | .53779 | .84308 | .55242 | .83356 | .56689 | .82380 | 28 |
| 33 | .50829 | .86119 | .52324 | .85218 | .53804 | .84292 | .55266 | .83340 | .56713 | .82363 | 27 |
| 34 | .50854 | .86104 | .52349 | .85203 | .53828 | .84277 | .55291 | .83324 | .56736 | .82347 | 26 |
| 35 | .50879 | .86089 | .52374 | .85188 | .53853 | .84261 | .55315 | .83308 | .56760 | .82330 | 25 |
| 36 | .50904 | .86074 | .52399 | .85173 | .53877 | .84245 | .55339 | .83292 | .56784 | .82314 | 24 |
| 37 | .50929 | .86059 | .52423 | .85157 | .53902 | .84230 | .55363 | .83276 | .56808 | .82297 | 23 |
| 38 | .50954 | .86045 | .52448 | .85142 | .53926 | .84214 | .55388 | .83260 | .56832 | .82281 | 22 |
| 39 | .50979 | .86030 | .52473 | .85127 | .53951 | .84198 | .55412 | .83244 | .56856 | .82264 | 21 |
| 40 | .51004 | .86015 | .52498 | .85112 | .53975 | .84183 | .55436 | .83228 | .56880 | .82248 | 20 |
| 41 | .51029 | .86000 | .52522 | .85096 | .54000 | .84167 | .55460 | .83212 | .56904 | .82231 | 19 |
| 42 | .51054 | .85985 | .52547 | .85081 | .54024 | .84151 | .55484 | .83195 | .56928 | .82214 | 18 |
| 43 | .51079 | .85970 | .52572 | .85066 | .54049 | .84135 | .55509 | .83179 | .56952 | .82198 | 17 |
| 44 | .51104 | .85956 | .52597 | .85051 | .54073 | .84120 | .55533 | .83163 | .56976 | .82181 | 16 |
| 45 | .51129 | .85941 | .52621 | .85035 | .54097 | .84104 | .55557 | .83147 | .57000 | .82163 | 15 |
| 46 | .51154 | .85926 | .52646 | .85020 | .54122 | .84088 | .55581 | .83131 | .57024 | .82148 | 14 |
| 47 | .51179 | .85911 | .52671 | .85005 | .54146 | .84072 | .55605 | .83115 | .57047 | .82132 | 13 |
| 48 | .51204 | .85896 | .52696 | .84989 | .54171 | .84057 | .55630 | .83098 | .57071 | .82115 | 12 |
| 49 | .51229 | .85881 | .52720 | .84974 | .54195 | .84041 | .55654 | .83082 | .57095 | .82098 | 11 |
| 50 | .51254 | .85866 | .52745 | .84959 | .54220 | .84025 | .55678 | .83066 | .57119 | .82082 | 10 |
| 51 | .51279 | .85851 | .52770 | .84943 | .54244 | .84009 | .55702 | .83050 | .57143 | .82065 | 9 |
| 52 | .51304 | .85836 | .52794 | .84928 | .54269 | .83994 | .55726 | .83034 | .57167 | .82048 | 8 |
| 53 | .51329 | .85821 | .52819 | .84913 | .54293 | .83978 | .55750 | .83017 | .57191 | .82032 | 7 |
| 54 | .51354 | .85806 | .52844 | .84897 | .54317 | .83962 | .55775 | .83001 | .57215 | .82015 | 6 |
| 55 | .51379 | .85792 | .52869 | .84882 | .54342 | .83946 | .55799 | .82985 | .57238 | .81999 | 5 |
| 56 | .51404 | .85777 | .52893 | .84866 | .54366 | .83930 | .55823 | .82969 | .57262 | .81982 | 4 |
| 57 | .51429 | .85762 | .52918 | .84851 | .54391 | .83915 | .55847 | .82953 | .57286 | .81965 | 3 |
| 58 | .51454 | .85747 | .52943 | .84836 | .54415 | .83909 | .55871 | .82936 | .57310 | .81949 | 2 |
| 59 | .51479 | .85732 | .52967 | .84820 | .54446 | .83893 | .55895 | .82920 | .57334 | .81932 | 1 |
| 60 | .51504 | .85717 | .52992 | .84805 | .54464 | .83807 | .55919 | .82904 | .57358 | .81915 | 0 |
| | Cosin | Sine | |

| 35° | | 36° | | 37° | | 38° | | 39° | | | |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | | |
| 0 | .57358 | .81915 | .58779 | .80902 | .60182 | .78654 | .61566 | .78801 | .62932 | .77715 | 60 |
| 1 | .57381 | .81899 | .58802 | .80885 | .60205 | .78546 | .61589 | .78783 | .62955 | .77696 | 59 |
| 2 | .57405 | .81882 | .58826 | .80867 | .60228 | .78529 | .61612 | .78765 | .62977 | .77678 | 58 |
| 3 | .57429 | .81865 | .58849 | .80850 | .60251 | .78511 | .61635 | .78747 | .63000 | .77660 | 57 |
| 4 | .57453 | .81848 | .58873 | .80833 | .60274 | .78493 | .61658 | .78729 | .63022 | .77641 | 56 |
| 5 | .57477 | .81832 | .58896 | .80816 | .60298 | .78476 | .61681 | .78711 | .63045 | .77623 | 55 |
| 6 | .57501 | .81815 | .58920 | .80799 | .60321 | .78458 | .61704 | .78694 | .63068 | .77605 | 54 |
| 7 | .57524 | .81798 | .58943 | .80782 | .60344 | .78441 | .61726 | .78676 | .63090 | .77586 | 53 |
| 8 | .57548 | .81782 | .58967 | .80765 | .60367 | .78423 | .61749 | .78658 | .63113 | .77568 | 52 |
| 9 | .57572 | .81765 | .58990 | .80748 | .60390 | .78405 | .61772 | .78640 | .63135 | .77550 | 51 |
| 10 | .57596 | .81748 | .59014 | .80730 | .60414 | .78388 | .61795 | .78622 | .63158 | .77531 | 50 |
| 11 | .57619 | .81731 | .59037 | .80713 | .60437 | .78371 | .61818 | .78604 | .63180 | .77515 | 49 |
| 12 | .57643 | .81714 | .59061 | .80696 | .60460 | .78353 | .61841 | .78586 | .63203 | .77494 | 48 |
| 13 | .57667 | .81698 | .59084 | .80679 | .60483 | .78335 | .61864 | .78568 | .63225 | .77476 | 47 |
| 14 | .57691 | .81681 | .59108 | .80662 | .60506 | .78318 | .61887 | .78550 | .63248 | .77458 | 46 |
| 15 | .57715 | .81664 | .59131 | .80644 | .60529 | .78300 | .61909 | .78532 | .63271 | .77439 | 45 |
| 16 | .57738 | .81647 | .59154 | .80627 | .60553 | .78283 | .61932 | .78514 | .63293 | .77421 | 44 |
| 17 | .57762 | .81631 | .59178 | .80610 | .60576 | .78265 | .61955 | .78496 | .63316 | .77402 | 43 |
| 18 | .57786 | .81614 | .59201 | .80593 | .60599 | .78247 | .61978 | .78478 | .63338 | .77384 | 42 |
| 19 | .57810 | .81597 | .59225 | .80576 | .60623 | .78230 | .62001 | .78460 | .63361 | .77366 | 41 |
| 20 | .57833 | .81580 | .59248 | .80558 | .60645 | .78212 | .62024 | .78442 | .63383 | .77347 | 40 |
| 21 | .57857 | .81563 | .59271 | .80541 | .60668 | .78194 | .62046 | .78424 | .63406 | .77329 | 39 |
| 22 | .57881 | .81546 | .59295 | .80524 | .60691 | .78176 | .62069 | .78405 | .63428 | .77310 | 38 |
| 23 | .57904 | .81530 | .59318 | .80507 | .60714 | .78159 | .62092 | .78387 | .63451 | .77292 | 37 |
| 24 | .57928 | .81513 | .59342 | .80489 | .60738 | .78141 | .62115 | .78369 | .63473 | .77273 | 36 |
| 25 | .57952 | .81496 | .59365 | .80472 | .60761 | .78124 | .62138 | .78351 | .63496 | .77255 | 35 |
| 26 | .57976 | .81479 | .59389 | .80455 | .60784 | .78106 | .62160 | .78333 | .63518 | .77236 | 34 |
| 27 | .57999 | .81462 | .59412 | .80438 | .60807 | .78088 | .62183 | .78315 | .63540 | .77218 | 33 |
| 28 | .58023 | .81445 | .59436 | .80420 | .60830 | .78071 | .62206 | .78297 | .63562 | .77199 | 32 |
| 29 | .58047 | .81428 | .59459 | .80403 | .60853 | .78053 | .62229 | .78279 | .63585 | .77181 | 31 |
| 30 | .58070 | .81412 | .59483 | .80386 | .60876 | .78035 | .62251 | .78261 | .63608 | .77162 | 30 |
| 31 | .58094 | .81395 | .59506 | .80368 | .60899 | .78018 | .62274 | .78243 | .63630 | .77144 | 29 |
| 32 | .58118 | .81378 | .59529 | .80351 | .60923 | .78000 | .62297 | .78225 | .63653 | .77125 | 28 |
| 33 | .58141 | .81361 | .59553 | .80334 | .60945 | .77982 | .62320 | .78196 | .63675 | .77107 | 27 |
| 34 | .58165 | .81344 | .59576 | .80316 | .60968 | .77964 | .62342 | .78168 | .63698 | .77088 | 26 |
| 35 | .58189 | .81327 | .59599 | .80299 | .60991 | .77947 | .62365 | .78170 | .63720 | .77070 | 25 |
| 36 | .58212 | .81310 | .59622 | .80282 | .61015 | .77929 | .62388 | .78152 | .63742 | .77051 | 24 |
| 37 | .58236 | .81293 | .59646 | .80264 | .61038 | .77911 | .62411 | .78134 | .63765 | .77033 | 23 |
| 38 | .58260 | .81276 | .59669 | .80247 | .61061 | .77893 | .62433 | .78116 | .63787 | .77014 | 22 |
| 39 | .58283 | .81259 | .59693 | .80230 | .61084 | .77876 | .62456 | .78098 | .63810 | .76996 | 21 |
| 40 | .58307 | .81242 | .59716 | .80212 | .61107 | .77858 | .62479 | .78079 | .63832 | .76977 | 20 |
| 41 | .58330 | .81225 | .59739 | .80196 | .61120 | .77840 | .62502 | .78061 | .63854 | .76959 | 19 |
| 42 | .58354 | .81208 | .59763 | .80178 | .61153 | .77812 | .62524 | .78043 | .63877 | .76940 | 18 |
| 43 | .58378 | .81191 | .59786 | .80160 | .61176 | .77795 | .62547 | .78025 | .63899 | .76921 | 17 |
| 44 | .58401 | .81174 | .59809 | .80143 | .61199 | .77768 | .62570 | .78007 | .63922 | .76903 | 16 |
| 45 | .58425 | .81157 | .59832 | .80125 | .61222 | .77751 | .62592 | .77988 | .63944 | .76884 | 15 |
| 46 | .58449 | .81140 | .59856 | .80108 | .61245 | .77705 | .62615 | .77970 | .63966 | .76866 | 14 |
| 47 | .58472 | .81123 | .59879 | .80091 | .61268 | .77693 | .62638 | .77952 | .63989 | .76847 | 13 |
| 48 | .58496 | .81106 | .59902 | .80073 | .61291 | .77676 | .62660 | .77934 | .64011 | .76828 | 12 |
| 49 | .58519 | .81089 | .59926 | .80056 | .61314 | .77659 | .62683 | .77916 | .64033 | .76810 | 11 |
| 50 | .58543 | .81072 | .59949 | .80038 | .61337 | .77642 | .62706 | .77897 | .64056 | .76791 | 10 |
| 51 | .58667 | .81055 | .59972 | .80021 | .61360 | .77625 | .62728 | .77879 | .64078 | .76772 | 9 |
| 52 | .58590 | .81038 | .59995 | .80003 | .61383 | .77604 | .62751 | .77861 | .64100 | .76754 | 8 |
| 53 | .58614 | .81021 | .60019 | .79988 | .61400 | .77586 | .62774 | .77843 | .64123 | .76735 | 7 |
| 54 | .58637 | .81004 | .60042 | .79968 | .61423 | .77568 | .62796 | .77824 | .64145 | .76717 | 6 |
| 55 | .58661 | .80987 | .60065 | .79951 | .61451 | .77541 | .62819 | .77806 | .64167 | .76698 | 5 |
| 56 | .58684 | .80970 | .60089 | .79934 | .61474 | .77523 | .62842 | .77788 | .64190 | .76679 | 4 |
| 57 | .58708 | .80953 | .60112 | .79916 | .61497 | .77505 | .62864 | .77769 | .64212 | .76661 | 3 |
| 58 | .58731 | .80936 | .60135 | .79899 | .61520 | .77487 | .62887 | .77751 | .64234 | .76642 | 2 |
| 59 | .58755 | .80919 | .60158 | .79881 | .61543 | .77469 | .62909 | .77733 | .64256 | .76623 | 1 |
| 60 | .58779 | .80902 | .60182 | .79864 | .61566 | .77451 | .62932 | .77715 | .64279 | .76604 | 0 |
| | Cosin | Sine | |

| | 40° | | 41° | | 42° | | 43° | | 44° | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| | Sine | Cosin | |
| 0 | .64279 | .76604 | .65066 | .75471 | .66913 | .74814 | .68800 | .73135 | .69466 | .71934 | 60 |
| 1 | .64301 | .76586 | .65268 | .75452 | .66935 | .74935 | .68821 | .73116 | .69487 | .71914 | 59 |
| 2 | .64323 | .76567 | .65650 | .75433 | .66956 | .74926 | .68842 | .73096 | .69508 | .71894 | 58 |
| 3 | .64346 | .76548 | .65672 | .75414 | .66978 | .74926 | .68864 | .73076 | .69529 | .71873 | 57 |
| 4 | .64368 | .76530 | .65694 | .75395 | .66999 | .74937 | .68885 | .73056 | .69549 | .71853 | 56 |
| 5 | .64390 | .76511 | .65716 | .75375 | .67021 | .74917 | .68896 | .73036 | .69570 | .71833 | 55 |
| 6 | .64412 | .76492 | .65738 | .75356 | .67043 | .74918 | .68927 | .73016 | .69591 | .71813 | 54 |
| 7 | .64435 | .76473 | .65759 | .75337 | .67064 | .74918 | .68949 | .72996 | .69612 | .71792 | 53 |
| 8 | .64457 | .76455 | .65781 | .75318 | .67086 | .74919 | .68970 | .72976 | .69633 | .71772 | 52 |
| 9 | .64479 | .76433 | .65803 | .75299 | .67107 | .74919 | .68991 | .72957 | .69654 | .71752 | 51 |
| 10 | .64501 | .76417 | .65825 | .75280 | .67129 | .74920 | .68942 | .72937 | .69675 | .71732 | 50 |
| 11 | .64524 | .76398 | .65847 | .75261 | .67151 | .74910 | .68442 | .72917 | .69696 | .71711 | 49 |
| 12 | .64543 | .76379 | .65869 | .75241 | .67172 | .74908 | .68455 | .72897 | .69717 | .71691 | 48 |
| 13 | .64553 | .76351 | .65891 | .75222 | .67194 | .74901 | .68476 | .72877 | .69737 | .71671 | 47 |
| 14 | .64590 | .76342 | .65913 | .75203 | .67215 | .74904 | .68497 | .72857 | .69758 | .71650 | 46 |
| 15 | .64612 | .76323 | .65935 | .75184 | .67237 | .74902 | .68518 | .72837 | .69778 | .71630 | 45 |
| 16 | .64635 | .76304 | .65956 | .75165 | .67258 | .74902 | .68539 | .72817 | .69800 | .71610 | 44 |
| 17 | .64657 | .76285 | .65978 | .75146 | .67280 | .73983 | .68561 | .72797 | .69821 | .71590 | 43 |
| 18 | .64679 | .76267 | .66000 | .75128 | .67301 | .73963 | .68582 | .72777 | .69842 | .71569 | 42 |
| 19 | .64701 | .76248 | .66022 | .75107 | .67323 | .73944 | .68603 | .72757 | .69862 | .71549 | 41 |
| 20 | .64723 | .76229 | .66044 | .75088 | .67344 | .73924 | .68624 | .72737 | .69883 | .71529 | 40 |
| 21 | .64746 | .76210 | .66066 | .75069 | .67366 | .73904 | .68645 | .72717 | .69904 | .71508 | 39 |
| 22 | .64768 | .76192 | .66088 | .75050 | .67387 | .73885 | .68666 | .72697 | .69925 | .71488 | 38 |
| 23 | .64790 | .76173 | .66104 | .75030 | .67400 | .73865 | .68688 | .72677 | .69946 | .71468 | 37 |
| 24 | .64812 | .76154 | .66131 | .75011 | .67420 | .73840 | .68709 | .72657 | .69966 | .71447 | 36 |
| 25 | .64834 | .76135 | .66153 | .74992 | .67452 | .73823 | .68730 | .72637 | .69987 | .71427 | 35 |
| 26 | .64856 | .76116 | .66175 | .74973 | .67473 | .73803 | .68751 | .72617 | .70008 | .71407 | 34 |
| 27 | .64878 | .76097 | .66197 | .74953 | .67495 | .73787 | .68772 | .72597 | .70029 | .71386 | 33 |
| 28 | .64901 | .76078 | .66218 | .74934 | .67516 | .73767 | .68793 | .72577 | .70049 | .71366 | 32 |
| 29 | .64923 | .76059 | .66240 | .74915 | .67533 | .73747 | .68814 | .72557 | .70070 | .71345 | 31 |
| 30 | .64945 | .76041 | .66262 | .74896 | .67559 | .73728 | .68835 | .72537 | .70091 | .71325 | 30 |
| 31 | .64967 | .76022 | .66284 | .74876 | .67580 | .73703 | .68857 | .72517 | .70112 | .71305 | 29 |
| 32 | .64989 | .76003 | .66306 | .74857 | .67602 | .73688 | .68878 | .72497 | .70132 | .71284 | 28 |
| 33 | .65011 | .75984 | .66327 | .74838 | .67623 | .73669 | .68899 | .72477 | .70153 | .71264 | 27 |
| 34 | .65033 | .75965 | .66349 | .74818 | .67645 | .73649 | .68920 | .72457 | .70174 | .71243 | 26 |
| 35 | .65055 | .75945 | .66371 | .74799 | .67666 | .73629 | .68941 | .72437 | .70195 | .71223 | 25 |
| 36 | .65077 | .75927 | .66393 | .74780 | .67688 | .73610 | .68962 | .72417 | .70215 | .71203 | 24 |
| 37 | .65100 | .75908 | .66414 | .74761 | .67709 | .73589 | .68983 | .72397 | .70236 | .71182 | 23 |
| 38 | .65122 | .75889 | .66434 | .74741 | .67730 | .73570 | .69004 | .72377 | .70257 | .71162 | 22 |
| 39 | .65144 | .75870 | .66453 | .74722 | .67752 | .73551 | .69025 | .72357 | .70277 | .71141 | 21 |
| 40 | .65166 | .75851 | .66480 | .74703 | .67773 | .73531 | .69046 | .72337 | .70298 | .71121 | 20 |
| 41 | .65188 | .75832 | .66501 | .74683 | .67795 | .73511 | .69067 | .72317 | .70319 | .71100 | 19 |
| 42 | .65210 | .75813 | .66523 | .74664 | .67816 | .73491 | .69088 | .72297 | .70339 | .71080 | 18 |
| 43 | .65233 | .75794 | .66545 | .74644 | .67837 | .73472 | .69109 | .72277 | .70360 | .71059 | 17 |
| 44 | .65254 | .75775 | .66565 | .74625 | .67859 | .73452 | .69130 | .72257 | .70381 | .71039 | 16 |
| 45 | .65276 | .75756 | .66588 | .74606 | .67880 | .73432 | .69151 | .72236 | .70401 | .71019 | 15 |
| 46 | .65298 | .75738 | .66610 | .74588 | .67901 | .73413 | .69172 | .72216 | .70422 | .70998 | 14 |
| 47 | .65320 | .75719 | .66632 | .74567 | .67923 | .73393 | .69193 | .72196 | .70443 | .70978 | 13 |
| 48 | .65342 | .75700 | .66653 | .74548 | .67944 | .73373 | .69214 | .72176 | .70463 | .70957 | 12 |
| 49 | .65364 | .75680 | .66675 | .74528 | .67965 | .73353 | .69235 | .72156 | .70484 | .70937 | 11 |
| 50 | .65386 | .75661 | .66697 | .74509 | .67987 | .73333 | .69256 | .72136 | .70505 | .70916 | 10 |
| 51 | .65408 | .75642 | .66718 | .74489 | .68008 | .73314 | .69277 | .72116 | .70525 | .70896 | 9 |
| 52 | .65430 | .75623 | .66740 | .74470 | .68029 | .73294 | .69298 | .72095 | .70546 | .70875 | 8 |
| 53 | .65452 | .75604 | .66763 | .74451 | .68051 | .73274 | .69319 | .72075 | .70567 | .70855 | 7 |
| 54 | .65474 | .75585 | .66783 | .74431 | .68072 | .73254 | .69340 | .72055 | .70587 | .70834 | 6 |
| 55 | .65496 | .75566 | .66805 | .74412 | .68093 | .73234 | .69361 | .72035 | .70608 | .70813 | 5 |
| 56 | .65518 | .75547 | .66827 | .74392 | .68115 | .73215 | .69382 | .72015 | .70628 | .70798 | 4 |
| 57 | .65540 | .75528 | .66848 | .74373 | .68136 | .73195 | .69403 | .71995 | .70649 | .70772 | 3 |
| 58 | .65562 | .75509 | .66870 | .74352 | .68157 | .73175 | .69424 | .71974 | .70670 | .70752 | 2 |
| 59 | .65584 | .75490 | .66891 | .74334 | .68179 | .73155 | .69445 | .71954 | .70690 | .70731 | 1 |
| 60 | .65606 | .75471 | .66913 | .74314 | .68200 | .73135 | .69466 | .71934 | .70711 | .70711 | 0 |
| | Cosin | Sine | | Cosin | Sine | | Cosin | Sine | | Cosin | Sine |
| | 49° | 48° | | 47° | 46° | | 45° | | | | |

NATURAL TANGENTS AND COTANGENTS. 125

| 0° | | 1° | | 2° | | 3° | | | |
|--------|-----------|--------|---------|--------|---------|--------|---------|--------|------|
| Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Cotang | Tang |
| .00000 | Infinite. | .01746 | 57.2900 | .03492 | 28.6383 | .05241 | 19.0811 | 60 | |
| .00029 | 3437.75 | .01775 | 56.3506 | .03521 | 28.8994 | .05270 | 18.9755 | | |
| .00058 | 1718.87 | .01804 | 55.4415 | .03550 | 28.1664 | .05299 | 18.8711 | 58 | |
| .00087 | 1145.92 | .01833 | 54.5618 | .03579 | 27.9372 | .05328 | 18.7678 | | |
| .00116 | 859.436 | .01862 | 53.7086 | .03609 | 27.7117 | .05357 | 18.6656 | 56 | |
| .00145 | 687.549 | .01891 | 52.8821 | .03638 | 27.4899 | .05387 | 18.5645 | | |
| .00175 | 573.957 | .01920 | 52.0807 | .03667 | 27.2715 | .05416 | 18.4645 | 54 | |
| .00204 | 491.106 | .01949 | 51.3032 | .03696 | 27.0566 | .05445 | 18.3655 | | |
| .00233 | 429.718 | .01978 | 50.5483 | .03725 | 26.8450 | .05474 | 18.2677 | 52 | |
| .00262 | 381.971 | .02007 | 49.8157 | .03754 | 26.6367 | .05503 | 18.1708 | | |
| .00291 | 343.774 | .02036 | 49.1089 | .03783 | 26.4316 | .05533 | 18.0750 | 50 | |
| .00320 | 312.521 | .02066 | 48.4121 | .03812 | 26.2296 | .05562 | 17.9802 | | |
| .00349 | 276.478 | .02095 | 47.7395 | .03842 | 26.007 | .05591 | 17.8863 | 48 | |
| .00378 | 234.441 | .02124 | 47.0338 | .03871 | 25.8348 | .05620 | 17.7934 | | |
| .00407 | 215.533 | .02153 | 46.4489 | .03900 | 25.6418 | .05649 | 17.7015 | 46 | |
| .00436 | 229.103 | .02182 | 45.894 | .03929 | 25.4517 | .05678 | 17.6106 | | |
| .00465 | 214.838 | .02211 | 45.2261 | .03958 | 25.2644 | .05708 | 17.5206 | 44 | |
| .00494 | 202.219 | .02240 | 44.6986 | .03987 | 25.0798 | .05737 | 17.4314 | | |
| .00523 | 190.984 | .02269 | 44.0661 | .04016 | 24.8978 | .05766 | 17.3432 | 42 | |
| .00552 | 180.938 | .02298 | 43.5061 | .04046 | 24.7183 | .05795 | 17.2558 | | |
| .00581 | 171.885 | .02328 | 42.9641 | .04075 | 24.5418 | .05824 | 17.1668 | 40 | |
| .00610 | 163.700 | .02357 | 42.4335 | .04104 | 24.3675 | .05854 | 17.0887 | | |
| .00640 | 156.239 | .02386 | 41.9158 | .04133 | 24.1957 | .05883 | 16.9990 | 38 | |
| .00669 | 149.405 | .02415 | 41.4106 | .04162 | 24.0263 | .05912 | 16.9150 | | |
| .00698 | 143.237 | .02444 | 40.9174 | .04191 | 23.8593 | .05941 | 16.8319 | 36 | |
| .00727 | 137.507 | .02473 | 40.4588 | .04220 | 23.6945 | .05970 | 16.7496 | | |
| .00756 | 132.319 | .02502 | 39.9655 | .04250 | 23.5321 | .05999 | 16.6631 | 34 | |
| .00785 | 127.321 | .02531 | 39.5059 | .04279 | 23.3718 | .06029 | 16.5874 | | |
| .00814 | 122.774 | .02560 | 39.0568 | .04308 | 23.2137 | .06058 | 16.5075 | 32 | |
| .00844 | 118.640 | .02589 | 38.6177 | .04337 | 23.0577 | .06087 | 16.4288 | | |
| .00873 | 114.589 | .02619 | 38.1885 | .04366 | 22.9088 | .06116 | 16.3499 | 30 | |
| .00902 | 110.893 | .02648 | 37.7686 | .04395 | 22.7519 | .06145 | 16.2722 | | |
| .00931 | 107.426 | .02677 | 37.3579 | .04424 | 22.6020 | .06175 | 16.1952 | 28 | |
| .00960 | 104.171 | .02706 | 36.9560 | .04454 | 22.4541 | .06204 | 16.1190 | | |
| .00989 | 101.107 | .02735 | 36.5627 | .04483 | 22.3081 | .06233 | 16.0435 | 26 | |
| .01018 | 98.2179 | .02764 | 36.1776 | .04512 | 22.1040 | .06262 | 15.9687 | | |
| .01047 | 95.4895 | .02793 | 35.8006 | .04541 | 22.0217 | .06291 | 15.8945 | 24 | |
| .01076 | 92.9085 | .02822 | 35.4318 | .04570 | 21.8813 | .06321 | 15.8211 | | |
| .01105 | 90.4688 | .02851 | 35.0695 | .04599 | 21.7426 | .06350 | 15.7488 | 22 | |
| .01135 | 88.1436 | .02880 | 34.7151 | .04628 | 21.6056 | .06379 | 15.6763 | | |
| .01164 | 85.9398 | .02910 | 34.3878 | .04658 | 21.4704 | .06408 | 15.6048 | 20 | |
| .01193 | 83.8485 | .02939 | 34.0273 | .04687 | 21.3369 | .06437 | 15.5340 | | |
| .01222 | 81.8470 | .02968 | 33.6935 | .04716 | 21.2049 | .06467 | 15.4688 | 18 | |
| .01251 | 79.9434 | .02997 | 33.3663 | .04745 | 21.0747 | .06496 | 15.3948 | | |
| .01280 | 78.1263 | .03026 | 33.0452 | .04774 | 20.9460 | .06525 | 15.3254 | 16 | |
| .01309 | 76.3900 | .03055 | 32.7308 | .04808 | 20.8188 | .06554 | 15.2571 | | |
| .01338 | 74.7292 | .03084 | 32.4218 | .04843 | 20.6932 | .06584 | 15.1893 | 14 | |
| .01367 | 73.1390 | .03114 | 32.1181 | .04862 | 20.5691 | .06613 | 15.1229 | | |
| .01396 | 71.6151 | .03143 | 31.8205 | .04891 | 20.4465 | .06642 | 15.0557 | 12 | |
| .01425 | 70.1533 | .03172 | 31.5284 | .04920 | 20.3223 | .06671 | 14.9898 | | |
| .01455 | 68.7501 | .03201 | 31.2416 | .04949 | 20.2056 | .06700 | 14.9244 | 10 | |
| .01484 | 67.4019 | .03230 | 30.9599 | .04978 | 20.0873 | .06730 | 14.8596 | | |
| .01513 | 66.1055 | .03259 | 30.6933 | .05007 | 19.9702 | .06759 | 14.7954 | 8 | |
| .01542 | 64.8580 | .03288 | 30.4116 | .05037 | 19.8546 | .06788 | 14.7317 | | |
| .01571 | 63.6567 | .03317 | 30.1446 | .05066 | 19.7403 | .06817 | 14.6685 | 6 | |
| .01600 | 62.4992 | .03346 | 29.8828 | .05095 | 19.6273 | .06847 | 14.6059 | | |
| .01629 | 61.3829 | .03376 | 29.6245 | .05124 | 19.5156 | .06876 | 14.5438 | 4 | |
| .01658 | 60.3058 | .03405 | 29.3711 | .05153 | 19.4051 | .06905 | 14.4823 | | |
| .01687 | 59.2659 | .03434 | 29.1920 | .05182 | 19.2989 | .06934 | 14.4212 | 2 | |
| .01716 | 58.2613 | .03463 | 28.8771 | .05212 | 19.1879 | .06963 | 14.3607 | | |
| .01746 | 57.2900 | .03492 | 28.6363 | .05241 | 19.0811 | .06993 | 14.3007 | 0 | |
| Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |
| 89° | 88° | 87° | 86° | | | | | | |

126 NATURAL TANGENTS AND COTANGENTS.

| | 4° | | 5° | | 6° | | 7° | | |
|----|--------|---------|--------|---------|--------|---------|--------|---------|----|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .06998 | 14.3007 | .08749 | 11.4301 | .10510 | 9.51436 | .12278 | 8.14435 | 60 |
| 1 | .07023 | 14.2411 | .08778 | 11.3919 | .10540 | 9.48781 | .12308 | 8.12481 | 59 |
| 2 | .07051 | 14.1821 | .08807 | 11.3540 | .10569 | 9.46141 | .12338 | 8.10536 | 58 |
| 3 | .07080 | 14.1235 | .08837 | 11.3163 | .10599 | 9.43515 | .12367 | 8.08600 | 57 |
| 4 | .07110 | 14.0655 | .08866 | 11.2789 | .10628 | 9.40904 | .12397 | 8.06674 | 56 |
| 5 | .07139 | 14.0079 | .08895 | 11.2417 | .10657 | 9.38307 | .12426 | 8.04756 | 55 |
| 6 | .07168 | 13.9507 | .08925 | 11.2048 | .10687 | 9.35724 | .12456 | 8.02848 | 54 |
| 7 | .07197 | 13.8940 | .08954 | 11.1681 | .10716 | 9.33155 | .12485 | 8.00948 | 53 |
| 8 | .07227 | 13.8378 | .08983 | 11.1316 | .10746 | 9.30599 | .12515 | 7.99058 | 52 |
| 9 | .07256 | 13.7821 | .09013 | 11.0954 | .10775 | 9.28058 | .12544 | 7.97176 | 51 |
| 10 | .07285 | 13.7367 | .09042 | 11.0594 | .10805 | 9.25530 | .12574 | 7.95302 | 50 |
| 11 | .07314 | 13.6719 | .09071 | 11.0237 | .10834 | 9.23016 | .12603 | 7.93498 | 49 |
| 12 | .07344 | 13.6174 | .09101 | 10.9883 | .10863 | 9.20516 | .12633 | 7.91582 | 48 |
| 13 | .07373 | 13.5634 | .09130 | 10.9529 | .10893 | 9.18028 | .12663 | 7.89734 | 47 |
| 14 | .07402 | 13.5098 | .09159 | 10.9178 | .10922 | 9.15554 | .12692 | 7.87895 | 46 |
| 15 | .07431 | 13.4566 | .09189 | 10.8829 | .10952 | 9.13093 | .12722 | 7.86004 | 45 |
| 16 | .07461 | 13.4039 | .09218 | 10.8483 | .10981 | 9.10646 | .12751 | 7.84242 | 44 |
| 17 | .07490 | 13.3515 | .09247 | 10.8189 | .11010 | 9.08211 | .12781 | 7.82428 | 43 |
| 18 | .07519 | 13.2996 | .09277 | 10.7797 | .11040 | 9.05779 | .12810 | 7.80623 | 42 |
| 19 | .07548 | 13.2480 | .09306 | 10.7437 | .11070 | 9.03379 | .12840 | 7.78925 | 41 |
| 20 | .07578 | 13.1969 | .09335 | 10.7119 | .11099 | 9.00968 | .12869 | 7.77035 | 40 |
| 21 | .07607 | 13.1461 | .09365 | 10.6788 | .11128 | 8.98506 | .12899 | 7.75254 | 39 |
| 22 | .07636 | 13.0953 | .09394 | 10.6450 | .11158 | 8.96227 | .12929 | 7.73480 | 38 |
| 23 | .07665 | 13.0458 | .09423 | 10.6118 | .11187 | 8.93867 | .12958 | 7.71715 | 37 |
| 24 | .07695 | 12.9903 | .09453 | 10.5789 | .11217 | 8.91520 | .12988 | 7.69957 | 36 |
| 25 | .07724 | 12.9469 | .09483 | 10.5462 | .11246 | 8.89185 | .13017 | 7.68206 | 35 |
| 26 | .07753 | 12.8981 | .09511 | 10.5136 | .11276 | 8.86862 | .13047 | 7.66466 | 34 |
| 27 | .07782 | 12.8496 | .09541 | 10.4818 | .11305 | 8.84551 | .13076 | 7.64793 | 33 |
| 28 | .07812 | 12.8014 | .09570 | 10.4491 | .11335 | 8.82252 | .13106 | 7.63005 | 32 |
| 29 | .07841 | 12.7536 | .09600 | 10.4172 | .11364 | 8.79964 | .13136 | 7.61287 | 31 |
| 30 | .07870 | 12.7062 | .09629 | 10.3854 | .11394 | 8.77689 | .13165 | 7.59475 | 30 |
| 31 | .07899 | 12.6591 | .09658 | 10.3538 | .11423 | 8.75425 | .13195 | 7.57872 | 29 |
| 32 | .07928 | 12.6124 | .09688 | 10.3224 | .11452 | 8.73172 | .13224 | 7.56176 | 28 |
| 33 | .07958 | 12.5660 | .09717 | 10.2913 | .11482 | 8.70981 | .13254 | 7.54487 | 27 |
| 34 | .07987 | 12.5199 | .09746 | 10.2602 | .11511 | 8.68701 | .13284 | 7.52806 | 26 |
| 35 | .08017 | 12.4743 | .09776 | 10.2294 | .11541 | 8.66482 | .13313 | 7.51133 | 25 |
| 36 | .08046 | 12.4288 | .09805 | 10.1988 | .11570 | 8.64275 | .13343 | 7.49463 | 24 |
| 37 | .08075 | 12.3838 | .09834 | 10.1688 | .11600 | 8.62078 | .13372 | 7.47806 | 23 |
| 38 | .08104 | 12.3390 | .09864 | 10.1381 | .11629 | 8.59893 | .13402 | 7.46154 | 22 |
| 39 | .08134 | 12.2946 | .09893 | 10.1080 | .11659 | 8.57718 | .13432 | 7.44509 | 21 |
| 40 | .08163 | 12.2505 | .09923 | 10.0780 | .11688 | 8.55555 | .13461 | 7.42871 | 20 |
| 41 | .08192 | 12.2067 | .09952 | 10.0483 | .11718 | 8.53402 | .13491 | 7.41240 | 19 |
| 42 | .08221 | 12.1632 | .09981 | 10.0187 | .11747 | 8.51269 | .13521 | 7.39616 | 18 |
| 43 | .08251 | 12.1201 | .10011 | 9.98931 | .11777 | 8.49128 | .13550 | 7.37999 | 17 |
| 44 | .08280 | 12.0772 | .10040 | 9.96007 | .11806 | 8.47007 | .13580 | 7.36399 | 16 |
| 45 | .08309 | 12.0346 | .10069 | 9.93101 | .11836 | 8.44896 | .13609 | 7.34766 | 15 |
| 46 | .08339 | 11.9923 | .10099 | 9.90211 | .11865 | 8.42795 | .13639 | 7.33190 | 14 |
| 47 | .08368 | 11.9504 | .10128 | 9.87388 | .11895 | 8.40705 | .13669 | 7.31600 | 13 |
| 48 | .08397 | 11.9087 | .10158 | 9.84848 | .11924 | 8.38025 | .13698 | 7.30018 | 12 |
| 49 | .08427 | 11.8673 | .10187 | 9.81641 | .11954 | 8.36555 | .13728 | 7.28443 | 11 |
| 50 | .08456 | 11.8262 | .10216 | 9.78817 | .11983 | 8.34496 | .13758 | 7.26873 | 10 |
| 51 | .08485 | 11.7833 | .10246 | 9.76009 | .12018 | 8.32446 | .13787 | 7.25310 | 9 |
| 52 | .08514 | 11.7448 | .10275 | 9.73217 | .12049 | 8.30406 | .13817 | 7.23764 | 8 |
| 53 | .08544 | 11.7045 | .10305 | 9.70441 | .12072 | 8.28376 | .13846 | 7.22204 | 7 |
| 54 | .08573 | 11.6645 | .10334 | 9.67680 | .12101 | 8.26355 | .13876 | 7.20661 | 6 |
| 55 | .08602 | 11.6248 | .10363 | 9.64935 | .12131 | 8.24345 | .13906 | 7.19125 | 5 |
| 56 | .08632 | 11.5853 | .10393 | 9.62205 | .12160 | 8.22344 | .13935 | 7.17594 | 4 |
| 57 | .08661 | 11.5461 | .10423 | 9.59490 | .12190 | 8.20352 | .13965 | 7.16071 | 3 |
| 58 | .08690 | 11.5073 | .10453 | 9.56791 | .12219 | 8.18370 | .13995 | 7.14553 | 2 |
| 59 | .08720 | 11.4683 | .10481 | 9.54106 | .12249 | 8.16398 | .14024 | 7.13043 | 1 |
| 60 | .08749 | 11.4301 | .10510 | 9.51436 | .12278 | 8.14435 | .14054 | 7.11587 | 0 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |

85°

84°

83°

82°

| | 8° | | 9° | | 10° | | 11° | | |
|----|--------|---------|--------|---------|--------|---------|--------|---------|----|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .14054 | 7.11537 | .15838 | 6.31875 | .17638 | 5.67128 | .19435 | 5.14455 | 60 |
| 1 | .14084 | 7.10038 | .15808 | 6.30189 | .17603 | 5.66165 | .19468 | 5.13658 | 59 |
| 2 | .14113 | 7.08546 | .15898 | 6.29007 | .17693 | 5.65205 | .19498 | 5.12862 | 58 |
| 3 | .14143 | 7.07059 | .15928 | 6.27829 | .17723 | 5.64248 | .19529 | 5.12069 | 57 |
| 4 | .14173 | 7.05379 | .15958 | 6.26655 | .17753 | 5.63295 | .19559 | 5.11279 | 56 |
| 5 | .14202 | 7.04105 | .15988 | 6.25496 | .17783 | 5.62344 | .19589 | 5.10490 | 55 |
| 6 | .14232 | 7.02637 | .16017 | 6.24321 | .17813 | 5.61397 | .19619 | 5.09704 | 54 |
| 7 | .14262 | 6.91174 | .16047 | 6.23160 | .17843 | 5.60452 | .19649 | 5.08921 | 53 |
| 8 | .14291 | 6.09718 | .16077 | 6.22003 | .17873 | 5.59511 | .19680 | 5.08139 | 52 |
| 9 | .14321 | 6.98208 | .16107 | 6.20851 | .17903 | 5.58573 | .19710 | 5.07360 | 51 |
| 10 | .14351 | 6.06828 | .16137 | 6.19708 | .17933 | 5.57638 | .19740 | 5.06584 | 50 |
| 11 | .14381 | 6.95385 | .16167 | 6.18559 | .17963 | 5.56706 | .19770 | 5.05809 | 49 |
| 12 | .14410 | 6.03032 | .16193 | 6.17419 | .17993 | 5.55777 | .19801 | 5.05037 | 48 |
| 13 | .14440 | 6.92353 | .16226 | 6.16283 | .18023 | 5.54851 | .19831 | 5.04267 | 47 |
| 14 | .14470 | 6.91104 | .16256 | 6.15151 | .18053 | 5.53927 | .19861 | 5.03499 | 46 |
| 15 | .14499 | 6.89338 | .16286 | 6.14023 | .18083 | 5.53007 | .19891 | 5.02734 | 45 |
| 16 | .14529 | 6.88278 | .16316 | 6.12809 | .18113 | 5.52000 | .19921 | 5.01971 | 44 |
| 17 | .14559 | 6.88374 | .16346 | 6.11779 | .18143 | 5.51176 | .19952 | 5.01210 | 43 |
| 18 | .14588 | 6.85475 | .16376 | 6.10664 | .18173 | 5.50264 | .19982 | 5.00451 | 42 |
| 19 | .14618 | 6.84032 | .16405 | 6.09552 | .18203 | 5.49356 | .20012 | 4.99695 | 41 |
| 20 | .14648 | 6.82694 | .16435 | 6.08444 | .18233 | 5.48451 | .20042 | 4.98940 | 40 |
| 21 | .14678 | 6.81912 | .16465 | 6.07340 | .18263 | 5.47548 | .20073 | 4.98188 | 39 |
| 22 | .14707 | 6.79388 | .16495 | 6.06240 | .18293 | 5.46648 | .20103 | 4.97488 | 38 |
| 23 | .14737 | 6.78504 | .16525 | 6.05143 | .18323 | 5.45751 | .20133 | 4.96690 | 37 |
| 24 | .14767 | 6.77199 | .16555 | 6.04051 | .18353 | 5.44857 | .20164 | 4.95945 | 36 |
| 25 | .14796 | 6.75398 | .16585 | 6.02962 | .18384 | 5.43903 | .20194 | 4.95201 | 35 |
| 26 | .14826 | 6.74403 | .16615 | 6.01878 | .18414 | 5.43077 | .20224 | 4.94460 | 34 |
| 27 | .14856 | 6.73188 | .16645 | 6.00797 | .18444 | 5.42193 | .20254 | 4.93721 | 33 |
| 28 | .14886 | 6.71789 | .16674 | 5.99720 | .18474 | 5.41309 | .20285 | 4.92984 | 32 |
| 29 | .14915 | 6.70450 | .16704 | 5.98646 | .18504 | 5.40429 | .20315 | 4.92249 | 31 |
| 30 | .14945 | 6.69116 | .16734 | 5.97576 | .18534 | 5.39552 | .20345 | 4.91516 | 30 |
| 31 | .14975 | 6.67787 | .16764 | 5.96510 | .18564 | 5.38677 | .20376 | 4.90785 | 29 |
| 32 | .15005 | 6.63468 | .16794 | 5.95448 | .18594 | 5.37805 | .20406 | 4.90056 | 28 |
| 33 | .15034 | 6.61544 | .16824 | 5.94390 | .18624 | 5.36988 | .20436 | 4.89380 | 27 |
| 34 | .15064 | 6.60831 | .16854 | 5.93865 | .18654 | 5.36070 | .20466 | 4.88605 | 26 |
| 35 | .15094 | 6.60232 | .16884 | 5.92988 | .18684 | 5.35208 | .20497 | 4.87829 | 25 |
| 36 | .15124 | 6.61219 | .16914 | 5.91236 | .18714 | 5.34345 | .20527 | 4.87162 | 24 |
| 37 | .15153 | 6.60221 | .16944 | 5.90101 | .18745 | 5.33487 | .20557 | 4.86444 | 23 |
| 38 | .15183 | 6.53027 | .16974 | 5.89151 | .18775 | 5.32631 | .20587 | 4.85727 | 22 |
| 39 | .15213 | 6.57339 | .17004 | 5.88114 | .18805 | 5.31778 | .20618 | 4.85018 | 21 |
| 40 | .15243 | 6.56055 | .17033 | 5.87080 | .18835 | 5.30928 | .20648 | 4.84300 | 20 |
| 41 | .15272 | 6.54777 | .17063 | 5.86051 | .18865 | 5.30080 | .20679 | 4.83590 | 19 |
| 42 | .15303 | 6.53533 | .17093 | 5.85024 | .18895 | 5.29235 | .20709 | 4.82882 | 18 |
| 43 | .15332 | 6.52224 | .17123 | 5.84001 | .18925 | 5.28393 | .20739 | 4.82175 | 17 |
| 44 | .15363 | 6.50970 | .17153 | 5.82982 | .18955 | 5.27553 | .20770 | 4.81471 | 16 |
| 45 | .15391 | 6.49710 | .17183 | 5.81066 | .18986 | 5.26715 | .20800 | 4.80769 | 15 |
| 46 | .15421 | 6.48456 | .17213 | 5.80933 | .19016 | 5.25330 | .20830 | 4.80068 | 14 |
| 47 | .15451 | 6.47206 | .17243 | 5.79944 | .19046 | 5.23048 | .20861 | 4.79370 | 13 |
| 48 | .15481 | 6.45961 | .17273 | 5.78938 | .19076 | 5.24218 | .20891 | 4.78673 | 12 |
| 49 | .15511 | 6.44790 | .17303 | 5.77936 | .19106 | 5.23301 | .20921 | 4.77978 | 11 |
| 50 | .15540 | 6.43484 | .17333 | 5.76937 | .19136 | 5.22566 | .20952 | 4.77286 | 10 |
| 51 | .15570 | 6.42253 | .17363 | 5.75941 | .19166 | 5.21744 | .20982 | 4.76595 | 9 |
| 52 | .15600 | 6.41026 | .17393 | 5.74049 | .19197 | 5.20925 | .21013 | 4.75906 | 8 |
| 53 | .15630 | 6.39804 | .17423 | 5.73060 | .19227 | 5.20107 | .21043 | 4.75219 | 7 |
| 54 | .15660 | 6.38574 | .17453 | 5.72074 | .19257 | 5.19223 | .21073 | 4.74534 | 6 |
| 55 | .15689 | 6.37374 | .17483 | 5.71092 | .19287 | 5.18400 | .21104 | 4.73851 | 5 |
| 56 | .15719 | 6.36155 | .17513 | 5.70113 | .19317 | 5.17671 | .21134 | 4.73170 | 4 |
| 57 | .15749 | 6.35001 | .17543 | 5.69037 | .19347 | 5.16863 | .21164 | 4.72490 | 3 |
| 58 | .15779 | 6.33761 | .17573 | 5.68064 | .19378 | 5.16058 | .21195 | 4.71813 | 2 |
| 59 | .15809 | 6.32566 | .17603 | 5.68094 | .19408 | 5.15256 | .21225 | 4.71187 | 1 |
| 60 | .15838 | 6.31375 | .17633 | 5.67128 | .19438 | 5.14456 | .21256 | 4.70463 | 0 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 81° | 80° | 79° | 78° | | | | | |

126 NATURAL TANGENTS AND COTANGENTS.

| | 4° | | 5° | | 6° | | 7° | | |
|----|--------|---------|--------|---------|--------|---------|--------|---------|----|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .06998 | 14.3007 | .08749 | 11.4301 | .10510 | 9.51436 | .12278 | 8.14435 | 60 |
| 1 | .07023 | 14.2411 | .08778 | 11.3919 | .10540 | 9.48781 | .12308 | 8.12481 | 59 |
| 2 | .07051 | 14.1821 | .08807 | 11.3540 | .10569 | 9.46141 | .12338 | 8.10538 | 58 |
| 3 | .07080 | 14.1235 | .08837 | 11.3163 | .10599 | 9.43515 | .12367 | 8.08600 | 57 |
| 4 | .07110 | 14.0655 | .08866 | 11.2789 | .10628 | 9.40904 | .12397 | 8.06674 | 56 |
| 5 | .07139 | 14.0079 | .08895 | 11.2417 | .10657 | 9.38307 | .12426 | 8.04756 | 55 |
| 6 | .07168 | 13.9507 | .08925 | 11.2048 | .10687 | 9.35724 | .12456 | 8.02848 | 54 |
| 7 | .07197 | 13.8940 | .08954 | 11.1681 | .10716 | 9.33155 | .12485 | 8.00948 | 53 |
| 8 | .07227 | 13.8378 | .08983 | 11.1316 | .10746 | 9.30599 | .12515 | 7.99058 | 52 |
| 9 | .07256 | 13.7821 | .09013 | 11.0954 | .10775 | 9.28058 | .12544 | 7.97176 | 51 |
| 10 | .07285 | 13.7267 | .09042 | 11.0594 | .10805 | 9.25530 | .12574 | 7.95802 | 50 |
| 11 | .07314 | 13.6719 | .09071 | 11.0237 | .10834 | 9.23016 | .12603 | 7.93488 | 49 |
| 12 | .07344 | 13.6174 | .09101 | 10.9882 | .10863 | 9.20516 | .12638 | 7.91582 | 48 |
| 13 | .07373 | 13.5634 | .09130 | 10.9529 | .10893 | 9.18028 | .12663 | 7.89734 | 47 |
| 14 | .07402 | 13.5098 | .09159 | 10.9178 | .10922 | 9.15554 | .12692 | 7.87985 | 46 |
| 15 | .07431 | 13.4566 | .09189 | 10.8829 | .10952 | 9.13093 | .12722 | 7.86064 | 45 |
| 16 | .07461 | 13.4039 | .09218 | 10.8483 | .10981 | 9.10646 | .12751 | 7.84242 | 44 |
| 17 | .07490 | 13.3515 | .09247 | 10.8139 | .11011 | 9.08211 | .12781 | 7.82428 | 43 |
| 18 | .07519 | 13.2996 | .09277 | 10.7797 | .11040 | 9.05719 | .12810 | 7.80622 | 42 |
| 19 | .07548 | 13.2480 | .09306 | 10.7437 | .11070 | 9.03379 | .12840 | 7.78925 | 41 |
| 20 | .07578 | 13.1969 | .09335 | 10.7119 | .11099 | 9.00983 | .12869 | 7.77035 | 40 |
| 21 | .07607 | 13.1461 | .09365 | 10.6783 | .11128 | 8.98598 | .12899 | 7.75254 | 39 |
| 22 | .07636 | 13.0958 | .09394 | 10.6450 | .11158 | 8.96227 | .12929 | 7.73480 | 38 |
| 23 | .07665 | 13.0458 | .09428 | 10.6118 | .11187 | 8.93867 | .12958 | 7.71715 | 37 |
| 24 | .07695 | 13.0002 | .09458 | 10.5789 | .11217 | 8.91520 | .12988 | 7.69357 | 36 |
| 25 | .07724 | 12.9469 | .09482 | 10.5462 | .11246 | 8.89185 | .13017 | 7.68206 | 35 |
| 26 | .07753 | 12.8981 | .09511 | 10.5136 | .11276 | 8.86862 | .13047 | 7.66468 | 34 |
| 27 | .07783 | 12.8490 | .09541 | 10.4818 | .11305 | 8.84551 | .13076 | 7.64738 | 33 |
| 28 | .07812 | 12.8014 | .09570 | 10.4491 | .11335 | 8.82253 | .13106 | 7.63005 | 32 |
| 29 | .07841 | 12.7536 | .09600 | 10.4173 | .11364 | 8.79964 | .13136 | 7.61287 | 31 |
| 30 | .07870 | 12.7062 | .09629 | 10.3854 | .11394 | 8.77689 | .13165 | 7.59575 | 30 |
| 31 | .07899 | 12.6591 | .09658 | 10.3538 | .11423 | 8.75425 | .13195 | 7.57872 | 29 |
| 32 | .07929 | 12.6124 | .09688 | 10.3224 | .11452 | 8.73172 | .13224 | 7.56176 | 28 |
| 33 | .07958 | 12.5606 | .09717 | 10.2913 | .11482 | 8.70981 | .13254 | 7.54487 | 27 |
| 34 | .07987 | 12.5199 | .09746 | 10.2602 | .11511 | 8.68701 | .13284 | 7.52806 | 26 |
| 35 | .08017 | 12.4742 | .09776 | 10.2294 | .11541 | 8.66482 | .13313 | 7.51132 | 25 |
| 36 | .08046 | 12.4288 | .09805 | 10.1988 | .11570 | 8.64275 | .13343 | 7.49465 | 24 |
| 37 | .08075 | 12.3888 | .09834 | 10.1683 | .11600 | 8.62078 | .13372 | 7.47806 | 23 |
| 38 | .08104 | 12.3390 | .09864 | 10.1381 | .11639 | 8.59893 | .13402 | 7.46154 | 22 |
| 39 | .08134 | 12.2946 | .09893 | 10.1080 | .11659 | 8.57718 | .13432 | 7.44509 | 21 |
| 40 | .08163 | 12.2505 | .09923 | 10.0780 | .11688 | 8.55555 | .13461 | 7.42871 | 20 |
| 41 | .08192 | 12.2067 | .09952 | 10.0483 | .11718 | 8.53402 | .13491 | 7.41240 | 19 |
| 42 | .08221 | 12.1633 | .09981 | 10.0187 | .11747 | 8.51269 | .13521 | 7.39616 | 18 |
| 43 | .08251 | 12.1201 | .10011 | 9.98931 | .11777 | 8.49128 | .13550 | 7.37998 | 17 |
| 44 | .08280 | 12.0772 | .10040 | 9.96007 | .11806 | 8.47007 | .13580 | 7.36392 | 16 |
| 45 | .08309 | 12.0346 | .10069 | 9.93101 | .11836 | 8.44806 | .13609 | 7.34786 | 15 |
| 46 | .08339 | 11.9923 | .10099 | 9.90211 | .11865 | 8.42795 | .13639 | 7.33190 | 14 |
| 47 | .08368 | 11.9504 | .10128 | 9.87388 | .11895 | 8.40705 | .13669 | 7.31600 | 13 |
| 48 | .08397 | 11.9087 | .10158 | 9.84482 | .11924 | 8.38625 | .13698 | 7.30018 | 12 |
| 49 | .08427 | 11.8673 | .10187 | 9.81641 | .11954 | 8.36555 | .13728 | 7.28442 | 11 |
| 50 | .08456 | 11.8262 | .10216 | 9.78817 | .11983 | 8.34496 | .13758 | 7.26873 | 10 |
| 51 | .08485 | 11.7853 | .10246 | 8.76009 | .12013 | 8.32446 | .13787 | 7.25310 | 9 |
| 52 | .08514 | 11.7448 | .10275 | 8.73217 | .12043 | 8.30406 | .13817 | 7.23754 | 8 |
| 53 | .08544 | 11.7045 | .10305 | 8.70441 | .12073 | 8.28376 | .13846 | 7.22204 | 7 |
| 54 | .08573 | 11.6645 | .10334 | 8.67680 | .12101 | 8.26355 | .13876 | 7.20661 | 6 |
| 55 | .08602 | 11.6248 | .10363 | 8.64935 | .12131 | 8.24345 | .13906 | 7.19125 | 5 |
| 56 | .08632 | 11.5853 | .10393 | 8.62205 | .12160 | 8.22344 | .13935 | 7.17594 | 4 |
| 57 | .08661 | 11.5461 | .10422 | 8.59490 | .12190 | 8.20352 | .13965 | 7.16071 | 3 |
| 58 | .08690 | 11.5072 | .10452 | 8.56791 | .12219 | 8.18370 | .13995 | 7.14553 | 2 |
| 59 | .08720 | 11.4685 | .10481 | 8.54106 | .12249 | 8.16398 | .14024 | 7.13042 | 1 |
| 60 | .08749 | 11.4301 | .10510 | 9.51436 | .12278 | 8.14435 | .14054 | 7.11537 | 0 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |

NATURAL TANGENTS AND COTANGENTS. 129

| 16° | | 17° | | 18° | | 19° | | |
|-----------|---------|--------|---------|--------|---------|--------|---------|------|
| Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 .29675 | 8.48741 | .30573 | 8.27085 | .32492 | 8.07768 | .34433 | 8.90421 | 60 |
| 1 .29706 | 8.48869 | .30605 | 8.26745 | .32524 | 8.07464 | .34465 | 8.90147 | 59 |
| 2 .29738 | 8.47977 | .30637 | 8.26406 | .32556 | 8.07160 | .34498 | 8.89873 | 58 |
| 3 .29769 | 8.47566 | .30669 | 8.26067 | .32588 | 8.06857 | .34530 | 8.89600 | 57 |
| 4 .29800 | 8.47216 | .30700 | 8.25729 | .32621 | 8.06554 | .34563 | 8.89327 | 56 |
| 5 .29832 | 8.46887 | .30732 | 8.25390 | .32653 | 8.06252 | .34596 | 8.89055 | 55 |
| 6 .29864 | 8.46458 | .30764 | 8.25055 | .32685 | 8.05950 | .34628 | 8.88783 | 54 |
| 7 .29895 | 8.46020 | .30796 | 8.24719 | .32717 | 8.05649 | .34661 | 8.88511 | 53 |
| 8 .29927 | 8.45703 | .30828 | 8.24383 | .32749 | 8.05349 | .34693 | 8.88240 | 52 |
| 9 .29958 | 8.45327 | .30860 | 8.24049 | .32782 | 8.05049 | .34726 | 8.87970 | 51 |
| 10 .29990 | 8.44951 | .30891 | 8.23714 | .32814 | 8.04749 | .34758 | 8.87700 | 50 |
| 11 .29021 | 8.44576 | .30923 | 8.23381 | .32846 | 8.04450 | .34791 | 8.87430 | 49 |
| 12 .29053 | 8.44202 | .30955 | 8.23048 | .32878 | 8.04152 | .34824 | 8.87161 | 48 |
| 13 .29084 | 8.43829 | .30987 | 8.22715 | .32911 | 8.03854 | .34856 | 8.86893 | 47 |
| 14 .29116 | 8.43456 | .31019 | 8.22384 | .32943 | 8.03556 | .34889 | 8.86624 | 46 |
| 15 .29147 | 8.43084 | .31051 | 8.22053 | .32975 | 8.03260 | .34923 | 8.86356 | 45 |
| 16 .29179 | 8.42713 | .31083 | 8.21722 | .33007 | 8.02963 | .34954 | 8.86089 | 44 |
| 17 .29210 | 8.42343 | .31115 | 8.21392 | .33040 | 8.02667 | .34987 | 8.85823 | 43 |
| 18 .29242 | 8.41973 | .31147 | 8.21063 | .33072 | 8.02373 | .35020 | 8.85558 | 42 |
| 19 .29274 | 8.41604 | .31178 | 8.20734 | .33104 | 8.02077 | .35053 | 8.85289 | 41 |
| 20 .29305 | 8.41236 | .31210 | 8.20406 | .33136 | 8.01783 | .35085 | 8.85023 | 40 |
| 21 .29337 | 8.40869 | .31242 | 8.20079 | .33169 | 8.01489 | .35118 | 8.84758 | 39 |
| 22 .29368 | 8.40502 | .31274 | 8.19752 | .33201 | 8.01196 | .35150 | 8.84494 | 38 |
| 23 .29400 | 8.40136 | .31306 | 8.19426 | .33233 | 8.00803 | .35182 | 8.84229 | 37 |
| 24 .29432 | 8.39771 | .31338 | 8.19100 | .33266 | 8.00011 | .35216 | 8.83965 | 36 |
| 25 .29463 | 8.39406 | .31370 | 8.18775 | .33298 | 8.00319 | .35248 | 8.83703 | 35 |
| 26 .29495 | 8.39043 | .31402 | 8.18451 | .33330 | 8.00028 | .35281 | 8.83438 | 34 |
| 27 .29526 | 8.38679 | .31434 | 8.18127 | .33363 | 8.99788 | .35314 | 8.83176 | 33 |
| 28 .29558 | 8.38317 | .31466 | 8.17804 | .33395 | 8.99447 | .35346 | 8.82914 | 32 |
| 29 .29590 | 8.37955 | .31498 | 8.17481 | .33427 | 8.99158 | .35379 | 8.82653 | 31 |
| 30 .29621 | 8.37594 | .31530 | 8.17159 | .33460 | 8.98868 | .35412 | 8.82391 | 30 |
| 31 .29653 | 8.37234 | .31562 | 8.16838 | .33492 | 8.98580 | .35445 | 8.82130 | 29 |
| 32 .29685 | 8.36875 | .31594 | 8.16517 | .33524 | 8.98292 | .35477 | 8.81870 | 28 |
| 33 .29716 | 8.36516 | .31626 | 8.16197 | .33557 | 8.98004 | .35510 | 8.81610 | 27 |
| 34 .29748 | 8.36158 | .31658 | 8.15877 | .33589 | 8.97717 | .35543 | 8.81350 | 26 |
| 35 .29780 | 8.35800 | .31690 | 8.15558 | .33621 | 8.97430 | .35576 | 8.81091 | 25 |
| 36 .29811 | 8.35443 | .31722 | 8.15240 | .33654 | 8.97144 | .35608 | 8.80833 | 24 |
| 37 .29843 | 8.35087 | .31754 | 8.14923 | .33686 | 8.96858 | .35641 | 8.80574 | 23 |
| 38 .29875 | 8.34733 | .31786 | 8.14605 | .33718 | 8.96573 | .35674 | 8.80316 | 22 |
| 39 .29906 | 8.34377 | .31818 | 8.14288 | .33751 | 8.96288 | .35707 | 8.80059 | 21 |
| 40 .29938 | 8.34033 | .31850 | 8.13973 | .33783 | 8.96004 | .35740 | 8.79803 | 20 |
| 41 .29970 | 8.33770 | .31882 | 8.18656 | .33816 | 8.95721 | .35772 | 8.79545 | 19 |
| 42 .30002 | 8.33517 | .31914 | 8.18341 | .33848 | 8.95437 | .35805 | 8.79380 | 18 |
| 43 .30033 | 8.33265 | .31946 | 8.18027 | .33881 | 8.95155 | .35838 | 8.79033 | 17 |
| 44 .30065 | 8.32914 | .31978 | 8.17718 | .33913 | 8.94873 | .35871 | 8.78778 | 16 |
| 45 .30097 | 8.32664 | .32010 | 8.17400 | .33945 | 8.94591 | .35904 | 8.78523 | 15 |
| 46 .30128 | 8.31914 | .32043 | 8.17087 | .33978 | 8.94309 | .35937 | 8.78269 | 14 |
| 47 .30160 | 8.31565 | .32074 | 8.11775 | .34010 | 8.94028 | .35969 | 8.78014 | 13 |
| 48 .30192 | 8.31216 | .32106 | 8.11464 | .34043 | 8.93748 | .36002 | 8.77761 | 12 |
| 49 .30224 | 8.30868 | .32139 | 8.11153 | .34075 | 8.93468 | .36035 | 8.77507 | 11 |
| 50 .30255 | 8.30521 | .32171 | 8.10843 | .34108 | 8.93189 | .36068 | 8.77254 | 10 |
| 51 .30287 | 8.30174 | .32203 | 8.10532 | .34140 | 8.92910 | .36101 | 8.77002 | 9 |
| 52 .30319 | 8.29829 | .32235 | 8.10223 | .34173 | 8.92632 | .36134 | 8.76750 | 8 |
| 53 .30351 | 8.29483 | .32267 | 8.09914 | .34205 | 8.92354 | .36167 | 8.76498 | 7 |
| 54 .30388 | 8.29193 | .32299 | 8.09606 | .34238 | 8.92076 | .36199 | 8.76247 | 6 |
| 55 .30414 | 8.28795 | .32331 | 8.09298 | .34270 | 8.91799 | .36238 | 8.75996 | 5 |
| 56 .30446 | 8.28453 | .32363 | 8.08991 | .34303 | 8.91523 | .36265 | 8.75746 | 4 |
| 57 .30478 | 8.28109 | .32396 | 8.08685 | .34335 | 8.91246 | .36298 | 8.75496 | 3 |
| 58 .30509 | 8.27767 | .32428 | 8.08379 | .34368 | 8.90971 | .36331 | 8.75246 | 2 |
| 59 .30541 | 8.27426 | .32460 | 8.08073 | .34400 | 8.90696 | .36364 | 8.74997 | 1 |
| 60 .30573 | 8.27085 | .32492 | 8.07768 | .34433 | 8.90421 | .36397 | 8.74748 | 0 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |
| | 73° | | 72° | | 71° | | 70° | |

128 NATURAL TANGENTS AND COTANGENTS.

| | 12° | | 13° | | 14° | | 15° | | |
|----|--------|---------|--------|---------|--------|---------|--------|---------|----|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .21296 | 4.70468 | .23067 | 4.88148 | .24983 | 4.01078 | .26795 | 8.78305 | 60 |
| 1 | .21286 | 4.69791 | .23117 | 4.82978 | .24964 | 4.00583 | .26826 | 8.72771 | 59 |
| 2 | .21316 | 4.69121 | .23148 | 4.82001 | .24905 | 4.00086 | .26857 | 8.72388 | 58 |
| 3 | .21347 | 4.68453 | .23179 | 4.81430 | .25028 | 3.99592 | .26888 | 8.71907 | 57 |
| 4 | .21377 | 4.67786 | .23209 | 4.80860 | .25058 | 3.99099 | .26920 | 8.71476 | 56 |
| 5 | .21408 | 4.67121 | .23240 | 4.80291 | .25087 | 3.98607 | .26951 | 8.71046 | 55 |
| 6 | .21438 | 4.66458 | .23271 | 4.29724 | .25118 | 3.98117 | .26983 | 8.70616 | 54 |
| 7 | .21468 | 4.65797 | .23301 | 4.29159 | .25149 | 3.97627 | .27013 | 8.70188 | 53 |
| 8 | .21499 | 4.65138 | .23332 | 4.28594 | .25180 | 3.97139 | .27044 | 8.69761 | 52 |
| 9 | .21529 | 4.64480 | .23363 | 4.28032 | .25211 | 3.96651 | .27076 | 8.69335 | 51 |
| 10 | .21560 | 4.63825 | .23393 | 4.27471 | .25242 | 3.96165 | .27107 | 8.68909 | 50 |
| 11 | .21590 | 4.63171 | .23424 | 4.26911 | .25273 | 3.95680 | .27138 | 8.68485 | 49 |
| 12 | .21621 | 4.62518 | .23455 | 4.26359 | .25304 | 3.95196 | .27169 | 8.68001 | 48 |
| 13 | .21651 | 4.61868 | .23486 | 4.25795 | .25335 | 3.94713 | .27201 | 8.67308 | 47 |
| 14 | .21682 | 4.61219 | .23516 | 4.25239 | .25366 | 3.94232 | .27232 | 8.67217 | 46 |
| 15 | .21712 | 4.60573 | .23547 | 4.24685 | .25397 | 3.93751 | .27263 | 8.66796 | 45 |
| 16 | .21743 | 4.59927 | .23578 | 4.24132 | .25428 | 3.93271 | .27294 | 8.66376 | 44 |
| 17 | .21773 | 4.59388 | .23608 | 4.23580 | .25459 | 3.92793 | .27326 | 8.65957 | 43 |
| 18 | .21804 | 4.58841 | .23639 | 4.23030 | .25490 | 3.92316 | .27357 | 8.65538 | 42 |
| 19 | .21834 | 4.58001 | .23670 | 4.22481 | .25521 | 3.91839 | .27388 | 8.65121 | 41 |
| 20 | .21864 | 4.57363 | .23700 | 4.21933 | .25552 | 3.91364 | .27419 | 8.64705 | 40 |
| 21 | .21895 | 4.56798 | .23731 | 4.21367 | .25583 | 3.90890 | .27451 | 8.64289 | 39 |
| 22 | .21925 | 4.56091 | .23762 | 4.20842 | .25614 | 3.90417 | .27482 | 8.63874 | 38 |
| 23 | .21956 | 4.55458 | .23793 | 4.20394 | .25645 | 3.89845 | .27513 | 8.63401 | 37 |
| 24 | .21986 | 4.54826 | .23823 | 4.19756 | .25676 | 3.89474 | .27545 | 8.63048 | 36 |
| 25 | .22017 | 4.54196 | .23854 | 4.19215 | .25707 | 3.89004 | .27576 | 8.62636 | 35 |
| 26 | .22047 | 4.53568 | .23883 | 4.18675 | .25738 | 3.88536 | .27607 | 8.62224 | 34 |
| 27 | .22078 | 4.52941 | .23916 | 4.18137 | .25769 | 3.88068 | .27638 | 8.61814 | 33 |
| 28 | .22108 | 4.52316 | .23946 | 4.17600 | .25800 | 3.87601 | .27670 | 8.61405 | 32 |
| 29 | .22139 | 4.51693 | .23977 | 4.17064 | .25831 | 3.87136 | .27701 | 8.60906 | 31 |
| 30 | .22169 | 4.51071 | .24008 | 4.16530 | .25862 | 3.86671 | .27732 | 8.60588 | 30 |
| 31 | .22200 | 4.50451 | .24039 | 4.15997 | .25893 | 3.86208 | .27764 | 8.60181 | 29 |
| 32 | .22231 | 4.49838 | .24069 | 4.15463 | .25924 | 3.85745 | .27795 | 8.59775 | 28 |
| 33 | .22261 | 4.49215 | .24100 | 4.14934 | .25954 | 3.85294 | .27826 | 8.59370 | 27 |
| 34 | .22293 | 4.48600 | .24131 | 4.14405 | .25986 | 3.84824 | .27858 | 8.58906 | 26 |
| 35 | .22323 | 4.47986 | .24163 | 4.13877 | .26017 | 3.84384 | .27889 | 8.58562 | 25 |
| 36 | .22353 | 4.47374 | .24193 | 4.13350 | .26048 | 3.83906 | .27921 | 8.58160 | 24 |
| 37 | .22383 | 4.46764 | .24223 | 4.12833 | .26079 | 3.83449 | .27952 | 8.57758 | 23 |
| 38 | .22414 | 4.46155 | .24254 | 4.12301 | .26110 | 3.82992 | .27983 | 8.57327 | 22 |
| 39 | .22444 | 4.45548 | .24285 | 4.11778 | .26141 | 3.82537 | .28015 | 8.56897 | 21 |
| 40 | .22475 | 4.44943 | .24316 | 4.11256 | .26172 | 3.82063 | .28046 | 8.56567 | 20 |
| 41 | .22506 | 4.44338 | .24347 | 4.10736 | .26203 | 3.81680 | .28077 | 8.56159 | 19 |
| 42 | .22536 | 4.43735 | .24377 | 4.10216 | .26233 | 3.81177 | .28109 | 8.55761 | 18 |
| 43 | .22567 | 4.43134 | .24408 | 4.09699 | .26266 | 3.80726 | .28140 | 8.55394 | 17 |
| 44 | .22597 | 4.42534 | .24439 | 4.09183 | .26297 | 3.80276 | .28173 | 8.54998 | 16 |
| 45 | .22628 | 4.41936 | .24470 | 4.08666 | .26328 | 3.79827 | .28208 | 8.54573 | 15 |
| 46 | .22658 | 4.41340 | .24501 | 4.08153 | .26359 | 3.79378 | .28244 | 8.54179 | 14 |
| 47 | .22689 | 4.40745 | .24532 | 4.07639 | .26390 | 3.78931 | .28286 | 8.53785 | 13 |
| 48 | .22719 | 4.40152 | .24563 | 4.07127 | .26421 | 3.78485 | .28327 | 8.53393 | 12 |
| 49 | .22750 | 4.39560 | .24593 | 4.06616 | .26452 | 3.78040 | .28369 | 8.53001 | 11 |
| 50 | .22781 | 4.38969 | .24624 | 4.06107 | .26483 | 3.77595 | .28400 | 8.52609 | 10 |
| 51 | .22811 | 4.38381 | .24655 | 4.05599 | .26515 | 3.77152 | .28431 | 8.52219 | 9 |
| 52 | .22842 | 4.37793 | .24686 | 4.05092 | .26546 | 3.76709 | .28463 | 8.51829 | 8 |
| 53 | .22872 | 4.37207 | .24717 | 4.04586 | .26577 | 3.76268 | .28494 | 8.51441 | 7 |
| 54 | .22903 | 4.36623 | .24747 | 4.04081 | .26608 | 3.75828 | .28526 | 8.51063 | 6 |
| 55 | .22934 | 4.36040 | .24778 | 4.03578 | .26639 | 3.75388 | .28567 | 8.50666 | 5 |
| 56 | .22964 | 4.35459 | .24809 | 4.03076 | .26670 | 3.74950 | .28609 | 8.50279 | 4 |
| 57 | .22995 | 4.34879 | .24840 | 4.02574 | .26701 | 3.74512 | .28650 | 8.49894 | 3 |
| 58 | .23026 | 4.34300 | .24871 | 4.02074 | .26733 | 3.74075 | .28692 | 8.49509 | 2 |
| 59 | .23056 | 4.33723 | .24902 | 4.01576 | .26764 | 3.73640 | .28643 | 8.49125 | 1 |
| 60 | .23087 | 4.33148 | .24933 | 4.01078 | .26795 | 3.73205 | .28675 | 8.48741 | - |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 77° | | 76° | | 75° | | 74° | | |

NATURAL TANGENTS AND COTANGENTS.

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| 16° | | 17° | | 18° | | 19° | |
|--------|---------|--------|---------|--------|---------|--------|---------|
| Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |
| .28675 | 3.48741 | .30573 | 3.27085 | .32492 | 3.07768 | .34433 | 2.90421 |
| .28706 | 3.48359 | .30605 | 3.26745 | .32524 | 3.07464 | .34465 | 2.90147 |
| .28738 | 3.47977 | .30637 | 3.26406 | .32556 | 3.07160 | .34498 | 2.89873 |
| .28769 | 3.47596 | .30669 | 3.26067 | .32588 | 3.06857 | .34530 | 2.89600 |
| .28800 | 3.47216 | .30700 | 3.25729 | .32621 | 3.06554 | .34563 | 2.89327 |
| .28832 | 3.46837 | .30732 | 3.25392 | .32653 | 3.06252 | .34596 | 2.89055 |
| .28864 | 3.46458 | .30764 | 3.25055 | .32685 | 3.05950 | .34628 | 2.88783 |
| .28895 | 3.46080 | .30796 | 3.24719 | .32717 | 3.05649 | .34661 | 2.88511 |
| .28927 | 3.45703 | .30828 | 3.24383 | .32749 | 3.05349 | .34693 | 2.88240 |
| .28958 | 3.45327 | .30860 | 3.24049 | .32782 | 3.05049 | .34726 | 2.87970 |
| .29090 | 3.44951 | .30891 | 3.23714 | .32814 | 3.04749 | .34758 | 2.87700 |
| .29021 | 3.44576 | .30923 | 3.23381 | .32846 | 3.04450 | .34791 | 2.87430 |
| .29053 | 3.44203 | .30955 | 3.23048 | .32878 | 3.04152 | .34824 | 2.87161 |
| .29084 | 3.43829 | .30987 | 3.22715 | .32911 | 3.03854 | .34856 | 2.86892 |
| .29116 | 3.43456 | .31019 | 3.22384 | .32943 | 3.03556 | .34889 | 2.86624 |
| .29147 | 3.43084 | .31051 | 3.22053 | .32975 | 3.03260 | .34922 | 2.86356 |
| .29179 | 3.42713 | .31083 | 3.21723 | .33007 | 3.02963 | .34954 | 2.86080 |
| .29210 | 3.42343 | .31115 | 3.21392 | .33040 | 3.02667 | .34987 | 2.85823 |
| .29242 | 3.41973 | .31147 | 3.21063 | .33072 | 3.02372 | .35020 | 2.85555 |
| .29274 | 3.41604 | .31178 | 3.20734 | .33104 | 3.02077 | .35053 | 2.85289 |
| .29305 | 3.41236 | .31210 | 3.20406 | .33136 | 3.01783 | .35086 | 2.85023 |
| .29337 | 3.40869 | .31242 | 3.20079 | .33169 | 3.01489 | .35118 | 2.84758 |
| .29368 | 3.40503 | .31274 | 3.19752 | .33201 | 3.01196 | .35150 | 2.84404 |
| .29400 | 3.40136 | .31306 | 3.19426 | .33233 | 3.00803 | .35183 | 2.84229 |
| .29432 | 3.39771 | .31338 | 3.19100 | .33266 | 3.00611 | .35216 | 2.83965 |
| .29463 | 3.39406 | .31370 | 3.18775 | .33298 | 3.00319 | .35248 | 2.83702 |
| .29495 | 3.39043 | .31402 | 3.18451 | .33330 | 3.00028 | .35281 | 2.83439 |
| .29526 | 3.38679 | .31434 | 3.18127 | .33363 | 2.99738 | .35314 | 2.83176 |
| .29558 | 3.38317 | .31466 | 3.17804 | .33395 | 2.99447 | .35346 | 2.82914 |
| .29590 | 3.37953 | .31498 | 3.17481 | .33427 | 2.99158 | .35379 | 2.82658 |
| .29621 | 3.37594 | .31530 | 3.17159 | .33460 | 2.98868 | .35412 | 2.82391 |
| .29653 | 3.37234 | .31562 | 3.16888 | .33492 | 2.98580 | .35445 | 2.82130 |
| .29685 | 3.36875 | .31594 | 3.16517 | .33524 | 2.98292 | .35477 | 2.81870 |
| .29716 | 3.36516 | .31626 | 3.16197 | .33557 | 2.98004 | .35510 | 2.81610 |
| .29748 | 3.36158 | .31658 | 3.15877 | .33589 | 2.97717 | .35543 | 2.81350 |
| .29780 | 3.35800 | .31690 | 3.15558 | .33621 | 2.97430 | .35570 | 2.81091 |
| .29811 | 3.35443 | .31722 | 3.15240 | .33654 | 2.97144 | .35608 | 2.80833 |
| .29843 | 3.35087 | .31754 | 3.14923 | .33686 | 2.96858 | .35641 | 2.80574 |
| .29875 | 3.34793 | .31786 | 3.14605 | .33718 | 2.96578 | .35674 | 2.80316 |
| .29906 | 3.34477 | .31818 | 3.14288 | .33751 | 2.96298 | .35707 | 2.80059 |
| .29938 | 3.34038 | .31850 | 3.13973 | .33783 | 2.96004 | .35740 | 2.79803 |
| .29970 | 3.33670 | .31882 | 3.13656 | .33816 | 2.95721 | .35772 | 2.79545 |
| .30001 | 3.33317 | .31914 | 3.13341 | .33848 | 2.95437 | .35805 | 2.79239 |
| .30033 | 3.32965 | .31946 | 3.13027 | .33881 | 2.95155 | .35838 | 2.79003 |
| .30065 | 3.32614 | .31978 | 3.12713 | .33913 | 2.94872 | .35871 | 2.78778 |
| .30097 | 3.32264 | .32010 | 3.12400 | .33945 | 2.94591 | .35904 | 2.78523 |
| .30128 | 3.31914 | .32042 | 3.12087 | .33978 | 2.94309 | .35937 | 2.78209 |
| .30160 | 3.31565 | .32074 | 3.11775 | .34010 | 2.94028 | .35969 | 2.78014 |
| .30192 | 3.31216 | .32106 | 3.11464 | .34043 | 2.93748 | .36002 | 2.77761 |
| .30224 | 3.30868 | .32139 | 3.11153 | .34075 | 2.93468 | .36035 | 2.77507 |
| .30255 | 3.30521 | .32171 | 3.10843 | .34108 | 2.93189 | .36068 | 2.77254 |
| .30287 | 3.30174 | .32203 | 3.10583 | .34140 | 2.92910 | .36101 | 2.77003 |
| .30319 | 3.29829 | .32235 | 3.10223 | .34173 | 2.92633 | .36134 | 2.76750 |
| .30351 | 3.29483 | .32267 | 3.09914 | .34205 | 2.92354 | .36167 | 2.76498 |
| .30383 | 3.29189 | .32299 | 3.09606 | .34238 | 2.92076 | .36199 | 2.76247 |
| .30414 | 3.28795 | .32331 | 3.09296 | .34270 | 2.91799 | .36238 | 2.75996 |
| .30446 | 3.28483 | .32363 | 3.08991 | .34303 | 2.91523 | .36265 | 2.75746 |
| .30478 | 3.28109 | .32395 | 3.08685 | .34335 | 2.91246 | .36298 | 2.75496 |
| .30509 | 3.27767 | .32428 | 3.08379 | .34368 | 2.90971 | .36331 | 2.75246 |
| .30541 | 3.27426 | .32460 | 3.08073 | .34400 | 2.90696 | .36364 | 2.74997 |
| .30573 | 3.27065 | .32492 | 3.07768 | .34433 | 2.90421 | .36397 | 2.74748 |
| Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |

130 NATURAL TANGENTS AND COTANGENTS.

| 30° | | 21° | | 22° | | 23° | |
|-----------|---------|--------|---------|--------|---------|--------|---------|
| Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |
| .36397 | 2.74748 | .38886 | 2.60509 | .40403 | 2.47509 | .42444 | 2.35585 |
| 1 .36430 | 2.74499 | .38420 | 2.60283 | .40436 | 2.47303 | .42482 | 2.35395 |
| 2 .36463 | 2.74251 | .38453 | 2.60057 | .40470 | 2.47005 | .42516 | 2.35205 |
| 3 .36496 | 2.74004 | .38487 | 2.59831 | .40504 | 2.46888 | .42551 | 2.35015 |
| 4 .36529 | 2.73758 | .38520 | 2.59606 | .40538 | 2.46682 | .42585 | 2.34825 |
| 5 .36562 | 2.73509 | .38553 | 2.59381 | .40572 | 2.46476 | .42619 | 2.34636 |
| 6 .36595 | 2.73263 | .38587 | 2.59156 | .40606 | 2.46270 | .42654 | 2.34447 |
| 7 .36628 | 2.73017 | .38620 | 2.58932 | .40643 | 2.46065 | .42688 | 2.34258 |
| 8 .36661 | 2.72771 | .38654 | 2.58708 | .40674 | 2.45860 | .42722 | 2.34069 |
| 9 .36694 | 2.72526 | .38687 | 2.58484 | .40707 | 2.45655 | .42757 | 2.33881 |
| 10 .36727 | 2.72281 | .38721 | 2.58261 | .40741 | 2.45451 | .42791 | 2.33693 |
| 11 .36760 | 2.72036 | .38754 | 2.58038 | .40775 | 2.45246 | .42826 | 2.33506 |
| 12 .36793 | 2.71792 | .38787 | 2.57815 | .40809 | 2.45048 | .42860 | 2.33317 |
| 13 .36826 | 2.71548 | .38821 | 2.57593 | .40843 | 2.44839 | .42894 | 2.33130 |
| 14 .36859 | 2.71305 | .38854 | 2.57371 | .40877 | 2.44636 | .42929 | 2.32943 |
| 15 .36892 | 2.71062 | .38888 | 2.57150 | .40911 | 2.44438 | .42963 | 2.32756 |
| 16 .36925 | 2.70819 | .38921 | 2.56928 | .40945 | 2.44230 | .42998 | 2.32570 |
| 17 .36958 | 2.70577 | .38955 | 2.56707 | .40979 | 2.44027 | .43032 | 2.32388 |
| 18 .36991 | 2.70335 | .38988 | 2.56487 | .41013 | 2.43825 | .43067 | 2.32197 |
| 19 .37024 | 2.70094 | .39023 | 2.56266 | .41047 | 2.43628 | .43101 | 2.32012 |
| 20 .37057 | 2.69853 | .39055 | 2.56046 | .41081 | 2.43422 | .43136 | 2.31829 |
| 21 .37090 | 2.69612 | .39089 | 2.55827 | .41115 | 2.43220 | .43170 | 2.31641 |
| 22 .37123 | 2.69371 | .39123 | 2.55608 | .41149 | 2.43019 | .43205 | 2.31456 |
| 23 .37157 | 2.69131 | .39156 | 2.55389 | .41183 | 2.42819 | .43239 | 2.31271 |
| 24 .37190 | 2.68892 | .39190 | 2.55170 | .41217 | 2.42618 | .43274 | 2.31086 |
| 25 .37223 | 2.68653 | .39223 | 2.54952 | .41251 | 2.42418 | .43308 | 2.30908 |
| 26 .37256 | 2.68414 | .39257 | 2.54734 | .41285 | 2.42218 | .43343 | 2.30718 |
| 27 .37289 | 2.68175 | .39290 | 2.54516 | .41319 | 2.42019 | .43378 | 2.30534 |
| 28 .37322 | 2.67937 | .39324 | 2.54299 | .41353 | 2.41819 | .43412 | 2.30351 |
| 29 .37355 | 2.67700 | .39357 | 2.54082 | .41387 | 2.41620 | .43447 | 2.30167 |
| 30 .37388 | 2.67462 | .39391 | 2.53865 | .41421 | 2.41421 | .43481 | 2.29984 |
| 31 .37422 | 2.67225 | .39425 | 2.53648 | .41455 | 2.41223 | .43516 | 2.29801 |
| 32 .37455 | 2.66989 | .39458 | 2.53432 | .41490 | 2.41025 | .43550 | 2.29619 |
| 33 .37488 | 2.66752 | .39492 | 2.53217 | .41524 | 2.40827 | .43585 | 2.29437 |
| 34 .37521 | 2.66516 | .39526 | 2.53001 | .41558 | 2.40629 | .43620 | 2.29254 |
| 35 .37554 | 2.66281 | .39559 | 2.52786 | .41592 | 2.40433 | .43654 | 2.29073 |
| 36 .37588 | 2.66046 | .39593 | 2.52571 | .41626 | 2.40235 | .43689 | 2.28891 |
| 37 .37621 | 2.65811 | .39626 | 2.52357 | .41660 | 2.40038 | .43724 | 2.28710 |
| 38 .37654 | 2.65576 | .39660 | 2.52142 | .41694 | 2.39841 | .43758 | 2.28528 |
| 39 .37687 | 2.65342 | .39694 | 2.51929 | .41728 | 2.39645 | .43793 | 2.28348 |
| 40 .37720 | 2.65109 | .39727 | 2.51715 | .41763 | 2.39449 | .43828 | 2.28167 |
| 41 .37754 | 2.64875 | .39761 | 2.51502 | .41797 | 2.39253 | .43862 | 2.27987 |
| 42 .37787 | 2.64642 | .39795 | 2.51299 | .41831 | 2.39058 | .43897 | 2.27806 |
| 43 .37820 | 2.64410 | .39829 | 2.51076 | .41865 | 2.38863 | .43932 | 2.27626 |
| 44 .37853 | 2.64177 | .39863 | 2.50864 | .41899 | 2.38668 | .43966 | 2.27447 |
| 45 .37887 | 2.63945 | .39896 | 2.50652 | .41933 | 2.38473 | .44001 | 2.27267 |
| 46 .37920 | 2.63714 | .39930 | 2.50440 | .41968 | 2.38279 | .44036 | 2.27086 |
| 47 .37953 | 2.63483 | .39963 | 2.50229 | .42002 | 2.38084 | .44071 | 2.26909 |
| 48 .37986 | 2.63252 | .40097 | 2.50018 | .42036 | 2.37891 | .44105 | 2.26730 |
| 49 .38020 | 2.63021 | .40031 | 2.49807 | .42070 | 2.37697 | .44140 | 2.26553 |
| 50 .38053 | 2.62791 | .40065 | 2.49597 | .42105 | 2.37504 | .44175 | 2.26374 |
| 51 .38086 | 2.62561 | .40098 | 2.49386 | .42139 | 2.37311 | .44210 | 2.26196 |
| 52 .38120 | 2.62332 | .40132 | 2.49177 | .42173 | 2.37118 | .44244 | 2.26018 |
| 53 .38153 | 2.62103 | .40166 | 2.48967 | .42207 | 2.36925 | .44279 | 2.25840 |
| 54 .38186 | 2.61874 | .40200 | 2.48758 | .42242 | 2.36733 | .44314 | 2.25663 |
| 55 .38220 | 2.61646 | .40234 | 2.48549 | .42276 | 2.36541 | .44349 | 2.25486 |
| 56 .38253 | 2.61418 | .40267 | 2.48340 | .42310 | 2.36349 | .44384 | 2.25309 |
| 57 .38286 | 2.61190 | .40301 | 2.48132 | .42345 | 2.36158 | .44418 | 2.25133 |
| 58 .38320 | 2.60963 | .40335 | 2.47924 | .42379 | 2.35967 | .44453 | 2.24956 |
| 59 .38353 | 2.60736 | .40369 | 2.47716 | .42413 | 2.35776 | .44488 | 2.24780 |
| 60 .38386 | 2.60509 | .40403 | 2.47509 | .42447 | 2.35585 | .44528 | 2.24604 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | |

69°

68°

67°

66°

NATURAL TANGENTS AND COTANGENTS.

131

| 24° | | 25° | | 26° | | 27° | |
|-----------|---------|--------|---------|--------|---------|--------|---------|
| Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |
| 0 .44528 | 2.24604 | .46631 | 2.14451 | .48773 | 2.05090 | .50968 | 1.96261 |
| 1 .44558 | 2.24428 | .46666 | 2.14288 | .48809 | 2.04879 | .50989 | 1.96120 |
| 2 .44598 | 2.24252 | .46702 | 2.14125 | .48845 | 2.04728 | .51026 | 1.95979 |
| 3 .44627 | 2.24077 | .46737 | 2.13963 | .48881 | 2.04577 | .51063 | 1.95838 |
| 4 .44662 | 2.23902 | .46772 | 2.13801 | .48917 | 2.04426 | .51099 | 1.95698 |
| 5 .44697 | 2.23727 | .46808 | 2.13639 | .48953 | 2.04276 | .51136 | 1.95557 |
| 6 .44732 | 2.23553 | .46843 | 2.13477 | .48989 | 2.04125 | .51173 | 1.95417 |
| 7 .44767 | 2.23378 | .46879 | 2.13316 | .49026 | 2.03975 | .51209 | 1.95277 |
| 8 .44802 | 2.23204 | .46914 | 2.13154 | .49062 | 2.03825 | .51246 | 1.95137 |
| 9 .44837 | 2.23030 | .46950 | 2.12998 | .49098 | 2.03675 | .51283 | 1.94997 |
| 10 .44872 | 2.22857 | .46985 | 2.12838 | .49134 | 2.03526 | .51319 | 1.94858 |
| 11 .44907 | 2.22683 | .47021 | 2.12671 | .49170 | 2.03376 | .51356 | 1.94718 |
| 12 .44942 | 2.22510 | .47056 | 2.12511 | .49206 | 2.03227 | .51393 | 1.94579 |
| 13 .44977 | 2.22337 | .47092 | 2.12350 | .49242 | 2.03078 | .51430 | 1.94440 |
| 14 .45012 | 2.22164 | .47128 | 2.12190 | .49278 | 2.02929 | .51467 | 1.94301 |
| 15 .45047 | 2.21993 | .47163 | 2.12030 | .49315 | 2.02780 | .51503 | 1.94163 |
| 16 .45082 | 2.21819 | .47199 | 2.11871 | .49351 | 2.02631 | .51540 | 1.94023 |
| 17 .45117 | 2.21647 | .47234 | 2.11711 | .49387 | 2.02483 | .51577 | 1.93885 |
| 18 .45152 | 2.21475 | .47270 | 2.11552 | .49423 | 2.02335 | .51614 | 1.93745 |
| 19 .45187 | 2.21304 | .47305 | 2.11392 | .49459 | 2.02187 | .51651 | 1.93608 |
| 20 .45222 | 2.21132 | .47341 | 2.11233 | .49495 | 2.02039 | .51688 | 1.93470 |
| 21 .45257 | 2.20961 | .47377 | 2.11075 | .49532 | 2.01891 | .51724 | 1.93333 |
| 22 .45292 | 2.20790 | .47412 | 2.10916 | .49568 | 2.01743 | .51761 | 1.93195 |
| 23 .45327 | 2.20619 | .47448 | 2.10758 | .49604 | 2.01596 | .51798 | 1.93057 |
| 24 .45362 | 2.20449 | .47483 | 2.10600 | .49640 | 2.01449 | .51835 | 1.92920 |
| 25 .45397 | 2.20278 | .47519 | 2.10442 | .49677 | 2.01303 | .51873 | 1.92783 |
| 26 .45432 | 2.20108 | .47555 | 2.10284 | .49713 | 2.01155 | .51909 | 1.92645 |
| 27 .45467 | 2.19938 | .47590 | 2.10126 | .49749 | 2.01008 | .51946 | 1.92508 |
| 28 .45502 | 2.19769 | .47626 | 2.09969 | .49786 | 2.00862 | .51983 | 1.92371 |
| 29 .45538 | 2.19595 | .47662 | 2.09811 | .49823 | 2.00715 | .52020 | 1.92233 |
| 30 .45573 | 2.19430 | .47696 | 2.09654 | .49858 | 2.00569 | .52057 | 1.92098 |
| 31 .45608 | 2.19261 | .47733 | 2.09496 | .49894 | 2.00423 | .52094 | 1.91968 |
| 32 .45643 | 2.19090 | .47769 | 2.09341 | .49931 | 2.00277 | .52131 | 1.91826 |
| 33 .45678 | 2.18923 | .47805 | 2.09184 | .49967 | 2.00131 | .52168 | 1.91690 |
| 34 .45713 | 2.18755 | .47840 | 2.09028 | .50004 | 1.99986 | .52205 | 1.91554 |
| 35 .45748 | 2.18587 | .47876 | 2.08872 | .50040 | 1.99841 | .52243 | 1.91418 |
| 36 .45784 | 2.18419 | .47912 | 2.08716 | .50076 | 1.99695 | .52279 | 1.91282 |
| 37 .45819 | 2.18251 | .47948 | 2.08560 | .50113 | 1.99550 | .52316 | 1.91147 |
| 38 .45854 | 2.18084 | .47984 | 2.08405 | .50149 | 1.99406 | .52353 | 1.91018 |
| 39 .45889 | 2.17916 | .48019 | 2.08250 | .50185 | 1.99261 | .52390 | 1.90867 |
| 40 .45924 | 2.17749 | .48055 | 2.08094 | .50223 | 1.99116 | .52427 | 1.90741 |
| 41 .45960 | 2.17582 | .48091 | 2.07939 | .50258 | 1.98973 | .52464 | 1.90607 |
| 42 .46095 | 2.17416 | .48127 | 2.07785 | .50295 | 1.98828 | .52501 | 1.90473 |
| 43 .46030 | 2.17249 | .48163 | 2.07630 | .50331 | 1.98684 | .52538 | 1.90337 |
| 44 .46065 | 2.17083 | .48198 | 2.07476 | .50368 | 1.98540 | .52575 | 1.90203 |
| 45 .46101 | 2.16917 | .48234 | 2.07321 | .50404 | 1.98396 | .52613 | 1.90069 |
| 46 .46136 | 2.16751 | .48270 | 2.07167 | .50441 | 1.98253 | .52650 | 1.89935 |
| 47 .46171 | 2.16585 | .48306 | 2.07014 | .50477 | 1.98110 | .52687 | 1.89801 |
| 48 .46206 | 2.16420 | .48342 | 2.06860 | .50514 | 1.97966 | .52724 | 1.89667 |
| 49 .46243 | 2.16253 | .48378 | 2.06706 | .50550 | 1.97823 | .52761 | 1.89533 |
| 50 .46277 | 2.16090 | .48414 | 2.06553 | .50587 | 1.97681 | .52798 | 1.89400 |
| 51 .46312 | 2.15925 | .48450 | 2.06400 | .50628 | 1.97538 | .52836 | 1.89266 |
| 52 .46348 | 2.15760 | .48486 | 2.06247 | .50660 | 1.97395 | .52873 | 1.89133 |
| 53 .46383 | 2.15596 | .48521 | 2.06094 | .50696 | 1.97253 | .52910 | 1.89000 |
| 54 .46418 | 2.15432 | .48557 | 2.05942 | .50733 | 1.97111 | .52947 | 1.88867 |
| 55 .46454 | 2.15268 | .48593 | 2.05790 | .50769 | 1.96969 | .52985 | 1.88734 |
| 56 .46489 | 2.15104 | .48629 | 2.05637 | .50806 | 1.96827 | .53023 | 1.88603 |
| 57 .46525 | 2.14940 | .48665 | 2.05485 | .50843 | 1.96685 | .53059 | 1.88469 |
| 58 .46560 | 2.14777 | .48701 | 2.05333 | .50879 | 1.96544 | .53096 | 1.88337 |
| 59 .46595 | 2.14614 | .48737 | 2.05182 | .50916 | 1.96402 | .53134 | 1.88205 |
| 60 .46631 | 2.14451 | .48773 | 2.05030 | .50953 | 1.96261 | .53171 | 1.88073 |
| Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |

65°

64°

63°

62°

132 NATURAL TANGENTS AND COTANGENTS.

| | 28° | | 29° | | 30° | | 31° | | |
|----|--------|---------|--------|---------|--------|---------|--------|---------|----|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .53171 | 1.88073 | .55431 | 1.80405 | .57735 | 1.73205 | .60066 | 1.66428 | 60 |
| 1 | .53206 | 1.87941 | .55469 | 1.80281 | .57774 | 1.73089 | .60126 | 1.66318 | 59 |
| 2 | .53246 | 1.87809 | .55507 | 1.80158 | .57818 | 1.72973 | .60165 | 1.66309 | 58 |
| 3 | .53283 | 1.87677 | .55545 | 1.80034 | .57851 | 1.72857 | .60205 | 1.66099 | 57 |
| 4 | .53320 | 1.87546 | .55583 | 1.79911 | .57890 | 1.72741 | .60245 | 1.65990 | 56 |
| 5 | .53358 | 1.87415 | .55621 | 1.79788 | .57929 | 1.72625 | .60284 | 1.65881 | 55 |
| 6 | .53395 | 1.87283 | .55659 | 1.79665 | .57968 | 1.72509 | .60324 | 1.65772 | 54 |
| 7 | .53433 | 1.87153 | .55697 | 1.79542 | .58007 | 1.72393 | .60364 | 1.65663 | 53 |
| 8 | .53470 | 1.87021 | .55736 | 1.79419 | .58046 | 1.72278 | .60403 | 1.65554 | 52 |
| 9 | .53507 | 1.86891 | .55774 | 1.79296 | .58085 | 1.72163 | .60443 | 1.65445 | 51 |
| 10 | .53545 | 1.86760 | .55812 | 1.79174 | .58124 | 1.72047 | .60483 | 1.65337 | 50 |
| 11 | .53582 | 1.86630 | .55850 | 1.79051 | .58162 | 1.71929 | .60523 | 1.65229 | 49 |
| 12 | .53620 | 1.86499 | .55888 | 1.78929 | .58201 | 1.71817 | .60663 | 1.65120 | 48 |
| 13 | .53657 | 1.86369 | .55926 | 1.78807 | .58240 | 1.71702 | .60602 | 1.65011 | 47 |
| 14 | .53694 | 1.86239 | .55964 | 1.78685 | .58279 | 1.71588 | .60642 | 1.64908 | 46 |
| 15 | .53732 | 1.86109 | .56003 | 1.78563 | .58318 | 1.71473 | .60681 | 1.64798 | 45 |
| 16 | .53769 | 1.86079 | .56041 | 1.78441 | .58357 | 1.71358 | .60721 | 1.64687 | 44 |
| 17 | .53807 | 1.86050 | .56079 | 1.78319 | .58396 | 1.71244 | .60761 | 1.64579 | 43 |
| 18 | .53844 | 1.85970 | .56117 | 1.78196 | .58435 | 1.71129 | .60801 | 1.64471 | 42 |
| 19 | .53882 | 1.85591 | .56156 | 1.78077 | .58474 | 1.71015 | .60841 | 1.64368 | 41 |
| 20 | .53920 | 1.85463 | .56194 | 1.77955 | .58513 | 1.70901 | .60881 | 1.64258 | 40 |
| 21 | .53957 | 1.85333 | .56232 | 1.77834 | .58552 | 1.70787 | .60921 | 1.64148 | 39 |
| 22 | .53995 | 1.85204 | .56270 | 1.77713 | .58591 | 1.70673 | .60960 | 1.64041 | 38 |
| 23 | .54032 | 1.85075 | .56309 | 1.77592 | .58631 | 1.70560 | .61000 | 1.63984 | 37 |
| 24 | .54070 | 1.84946 | .56347 | 1.77471 | .58670 | 1.70446 | .61040 | 1.63886 | 36 |
| 25 | .54107 | 1.84818 | .56385 | 1.77351 | .58709 | 1.70332 | .61080 | 1.63719 | 35 |
| 26 | .54145 | 1.84689 | .56424 | 1.77230 | .58748 | 1.70219 | .61120 | 1.63619 | 34 |
| 27 | .54183 | 1.84561 | .56462 | 1.77110 | .58787 | 1.70106 | .61160 | 1.63508 | 33 |
| 28 | .54220 | 1.84433 | .56501 | 1.76990 | .58826 | 1.69992 | .61200 | 1.63398 | 32 |
| 29 | .54258 | 1.84305 | .56539 | 1.76869 | .58865 | 1.69879 | .61240 | 1.63292 | 31 |
| 30 | .54296 | 1.84177 | .56577 | 1.76749 | .58905 | 1.69766 | .61280 | 1.63188 | 30 |
| 31 | .54333 | 1.84049 | .56616 | 1.76629 | .58944 | 1.69653 | .61320 | 1.63079 | 29 |
| 32 | .54371 | 1.83923 | .56654 | 1.76510 | .58983 | 1.69541 | .61360 | 1.62973 | 28 |
| 33 | .54409 | 1.83794 | .56692 | 1.76390 | .59023 | 1.69428 | .61400 | 1.62866 | 27 |
| 34 | .54446 | 1.83667 | .56731 | 1.76271 | .59061 | 1.69316 | .61440 | 1.62760 | 26 |
| 35 | .54484 | 1.83540 | .56769 | 1.76151 | .59101 | 1.69208 | .61480 | 1.62654 | 25 |
| 36 | .54523 | 1.83412 | .56808 | 1.76032 | .59140 | 1.69091 | .61520 | 1.62548 | 24 |
| 37 | .54560 | 1.83286 | .56846 | 1.75913 | .59179 | 1.68979 | .61561 | 1.62448 | 23 |
| 38 | .54597 | 1.83159 | .56885 | 1.75794 | .59218 | 1.68866 | .61601 | 1.62336 | 22 |
| 39 | .54635 | 1.83033 | .56923 | 1.75675 | .59258 | 1.68754 | .61641 | 1.62230 | 21 |
| 40 | .54673 | 1.82906 | .56963 | 1.75556 | .59297 | 1.68643 | .61681 | 1.62125 | 20 |
| 41 | .54711 | 1.82780 | .57000 | 1.75437 | .59336 | 1.68531 | .61721 | 1.62019 | 19 |
| 42 | .54748 | 1.82654 | .57039 | 1.75319 | .59376 | 1.68419 | .61761 | 1.61914 | 18 |
| 43 | .54786 | 1.82528 | .57078 | 1.75200 | .59415 | 1.68306 | .61801 | 1.61808 | 17 |
| 44 | .54824 | 1.82402 | .57116 | 1.75083 | .59454 | 1.68196 | .61842 | 1.61708 | 16 |
| 45 | .54862 | 1.82276 | .57155 | 1.74964 | .59494 | 1.68086 | .61882 | 1.61698 | 15 |
| 46 | .54900 | 1.82150 | .57193 | 1.74846 | .59533 | 1.67974 | .61922 | 1.61493 | 14 |
| 47 | .54938 | 1.82025 | .57232 | 1.74728 | .59573 | 1.67863 | .61962 | 1.61388 | 13 |
| 48 | .54975 | 1.81909 | .57271 | 1.74610 | .59612 | 1.67752 | .62003 | 1.61288 | 12 |
| 49 | .55013 | 1.81774 | .57309 | 1.74492 | .59651 | 1.67641 | .62043 | 1.61179 | 11 |
| 50 | .55051 | 1.81649 | .57348 | 1.74375 | .59691 | 1.67530 | .62083 | 1.61074 | 10 |
| 51 | .55089 | 1.81524 | .57386 | 1.74257 | .59730 | 1.67419 | .62124 | 1.60970 | 9 |
| 52 | .55127 | 1.81399 | .57425 | 1.74140 | .59770 | 1.67309 | .62164 | 1.60865 | 8 |
| 53 | .55165 | 1.81274 | .57464 | 1.74022 | .59809 | 1.67198 | .62204 | 1.60761 | 7 |
| 54 | .55203 | 1.81150 | .57508 | 1.73895 | .59849 | 1.67088 | .62245 | 1.60657 | 6 |
| 55 | .55241 | 1.81028 | .57541 | 1.73768 | .59888 | 1.66978 | .62286 | 1.60553 | 5 |
| 56 | .55279 | 1.80901 | .57580 | 1.73671 | .59928 | 1.66867 | .62325 | 1.60449 | 4 |
| 57 | .55317 | 1.80777 | .57619 | 1.73555 | .59967 | 1.66757 | .62366 | 1.60345 | 3 |
| 58 | .55355 | 1.80653 | .57657 | 1.73438 | .60007 | 1.66647 | .62406 | 1.60241 | 2 |
| 59 | .55393 | 1.80529 | .57696 | 1.73321 | .60046 | 1.66538 | .62446 | 1.60137 | 1 |
| 60 | .55431 | 1.80406 | .57735 | 1.73206 | .60086 | 1.66428 | .62487 | 1.60038 | 0 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 61° | | 60° | | 59° | | 58° | | |

NATURAL TANGENTS AND COTANGENTS. 133

| 32° | | 33° | | 34° | | 35° | |
|-----------|---------|--------|---------|--------|---------|--------|---------|
| Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |
| 0 .62497 | 1.60088 | .64941 | 1.53986 | .67451 | 1.49856 | .70021 | 1.42815 |
| 1 .62527 | 1.59930 | .64969 | 1.53888 | .67498 | 1.48163 | .70064 | 1.42726 |
| 2 .62556 | 1.59838 | .65024 | 1.53791 | .67536 | 1.48070 | .70107 | 1.42838 |
| 3 .62586 | 1.59723 | .65065 | 1.53698 | .67575 | 1.47977 | .70151 | 1.42550 |
| 4 .62616 | 1.59620 | .65106 | 1.53595 | .67620 | 1.47885 | .70194 | 1.42462 |
| 5 .62646 | 1.59517 | .65148 | 1.53497 | .67663 | 1.47793 | .70238 | 1.42374 |
| 6 .62676 | 1.59414 | .65189 | 1.53400 | .67705 | 1.47699 | .70281 | 1.42286 |
| 7 .62707 | 1.59311 | .65231 | 1.53308 | .67748 | 1.47607 | .70323 | 1.42198 |
| 8 .62811 | 1.59208 | .65273 | 1.53205 | .67790 | 1.47514 | .70368 | 1.42110 |
| 9 .62853 | 1.59105 | .65314 | 1.53107 | .67833 | 1.47422 | .70412 | 1.42023 |
| 10 .62883 | 1.59003 | .65355 | 1.53010 | .67875 | 1.47330 | .70455 | 1.41934 |
| 11 .62938 | 1.58900 | .65397 | 1.52913 | .67917 | 1.47238 | .70499 | 1.41847 |
| 12 .62973 | 1.58797 | .65438 | 1.52816 | .67960 | 1.47146 | .70542 | 1.41759 |
| 13 .63014 | 1.58695 | .65480 | 1.52719 | .68003 | 1.47053 | .70586 | 1.41672 |
| 14 .63055 | 1.58593 | .65521 | 1.52623 | .68045 | 1.46962 | .70629 | 1.41584 |
| 15 .63095 | 1.58490 | .65563 | 1.52525 | .68088 | 1.46870 | .70673 | 1.41497 |
| 16 .63136 | 1.58388 | .65604 | 1.52429 | .65130 | 1.46778 | .70717 | 1.41409 |
| 17 .63177 | 1.58286 | .65646 | 1.52332 | .65173 | 1.46686 | .70760 | 1.41322 |
| 18 .63217 | 1.58184 | .65688 | 1.52235 | .65215 | 1.46595 | .70804 | 1.41235 |
| 19 .63258 | 1.58083 | .65729 | 1.52139 | .65258 | 1.46503 | .70848 | 1.41148 |
| 20 .63299 | 1.57981 | .65771 | 1.52043 | .65301 | 1.46411 | .70891 | 1.41061 |
| 21 .63340 | 1.57879 | .65813 | 1.51946 | .65343 | 1.46320 | .70935 | 1.40974 |
| 22 .63380 | 1.57773 | .65854 | 1.51850 | .65386 | 1.46229 | .70979 | 1.40887 |
| 23 .63421 | 1.57676 | .65896 | 1.51754 | .65424 | 1.46137 | .71028 | 1.40800 |
| 24 .63463 | 1.57575 | .65938 | 1.51658 | .65471 | 1.46046 | .71066 | 1.40714 |
| 25 .63505 | 1.57474 | .65980 | 1.51560 | .65514 | 1.45953 | .71110 | 1.40627 |
| 26 .63544 | 1.57372 | .66021 | 1.51466 | .65557 | 1.45864 | .71154 | 1.40540 |
| 27 .63584 | 1.57271 | .66063 | 1.51370 | .65600 | 1.45773 | .71198 | 1.40454 |
| 28 .63625 | 1.57170 | .66105 | 1.51275 | .65643 | 1.45683 | .71242 | 1.40367 |
| 29 .63666 | 1.57069 | .66147 | 1.51179 | .65685 | 1.45593 | .71285 | 1.40281 |
| 30 .63707 | 1.56968 | .66189 | 1.51084 | .65728 | 1.45501 | .71329 | 1.40195 |
| 31 .63748 | 1.56868 | .66230 | 1.50988 | .65771 | 1.45410 | .71373 | 1.40109 |
| 32 .63789 | 1.56767 | .66272 | 1.50888 | .65814 | 1.45320 | .71417 | 1.40023 |
| 33 .63830 | 1.56667 | .66314 | 1.50797 | .65857 | 1.45229 | .71461 | 1.39936 |
| 34 .63871 | 1.56566 | .66356 | 1.50703 | .65900 | 1.45139 | .71505 | 1.39850 |
| 35 .63912 | 1.56466 | .66398 | 1.50607 | .65942 | 1.45049 | .71549 | 1.39764 |
| 36 .63953 | 1.56368 | .66440 | 1.50512 | .65985 | 1.44958 | .71588 | 1.39679 |
| 37 .63994 | 1.56265 | .66482 | 1.50417 | .66028 | 1.44868 | .71637 | 1.39593 |
| 38 .64035 | 1.56165 | .66524 | 1.50323 | .66071 | 1.44778 | .71681 | 1.39507 |
| 39 .64076 | 1.56065 | .66566 | 1.50228 | .66114 | 1.44688 | .71725 | 1.39421 |
| 40 .64117 | 1.55966 | .66608 | 1.50138 | .66157 | 1.44598 | .71769 | 1.39336 |
| 41 .64158 | 1.55866 | .66650 | 1.50038 | .66200 | 1.44508 | .71813 | 1.39250 |
| 42 .64199 | 1.55766 | .66692 | 1.49944 | .66243 | 1.44418 | .71857 | 1.39165 |
| 43 .64240 | 1.55666 | .66734 | 1.49849 | .66286 | 1.44329 | .71901 | 1.39079 |
| 44 .64281 | 1.55567 | .66776 | 1.49755 | .66329 | 1.44239 | .71946 | 1.39094 |
| 45 .64323 | 1.55467 | .66818 | 1.49661 | .66372 | 1.44149 | .71990 | 1.38909 |
| 46 .64363 | 1.55368 | .66860 | 1.49566 | .66416 | 1.44060 | .72034 | 1.38824 |
| 47 .64404 | 1.55269 | .66902 | 1.49473 | .66459 | 1.43970 | .72078 | 1.38738 |
| 48 .64446 | 1.55170 | .66944 | 1.49378 | .66500 | 1.43881 | .72122 | 1.38653 |
| 49 .64487 | 1.55071 | .66986 | 1.49284 | .66545 | 1.43793 | .72167 | 1.38568 |
| 50 .64528 | 1.54973 | .67028 | 1.49190 | .66588 | 1.43703 | .72211 | 1.38484 |
| 51 .64569 | 1.54873 | .67071 | 1.49097 | .66631 | 1.43614 | .72255 | 1.38399 |
| 52 .64610 | 1.54774 | .67118 | 1.49008 | .66675 | 1.43525 | .72299 | 1.38314 |
| 53 .64653 | 1.54675 | .67155 | 1.48909 | .66718 | 1.43436 | .72344 | 1.38229 |
| 54 .64698 | 1.54576 | .67197 | 1.48816 | .66761 | 1.43347 | .72388 | 1.38145 |
| 55 .64734 | 1.54478 | .67239 | 1.48723 | .66804 | 1.43258 | .72438 | 1.38060 |
| 56 .64775 | 1.54379 | .67280 | 1.48629 | .66847 | 1.43169 | .72477 | 1.37976 |
| 57 .64817 | 1.54281 | .67324 | 1.48536 | .66891 | 1.43080 | .72521 | 1.37891 |
| 58 .64858 | 1.54183 | .67366 | 1.48443 | .66934 | 1.42993 | .72565 | 1.37807 |
| 59 .64899 | 1.54085 | .67409 | 1.48349 | .66977 | 1.42908 | .72610 | 1.37723 |
| 60 .64941 | 1.53986 | .67451 | 1.48256 | .70021 | 1.42815 | .72654 | 1.37638 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | |

134. NATURAL TANGENTS AND COTANGENTS.

| 36° | | 37° | | 38° | | 39° | |
|-----------|---------|--------|---------|--------|---------|--------|---------|
| Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |
| 0 .72654 | 1.37638 | .75355 | 1.32704 | .78129 | 1.27994 | .80978 | 1.23490 |
| 1 .72699 | 1.37554 | .75401 | 1.32624 | .78175 | 1.27917 | .81027 | 1.23416 |
| 2 .72748 | 1.37470 | .75447 | 1.32544 | .78222 | 1.27841 | .81075 | 1.23348 |
| 3 .72788 | 1.37386 | .75492 | 1.32464 | .78269 | 1.27764 | .81128 | 1.23270 |
| 4 .72832 | 1.37302 | .75538 | 1.32384 | .78316 | 1.27688 | .81171 | 1.23194 |
| 5 .72877 | 1.37218 | .75584 | 1.32304 | .78363 | 1.27611 | .81220 | 1.23128 |
| 6 .72921 | 1.37134 | .75629 | 1.32224 | .78410 | 1.27535 | .81268 | 1.23050 |
| 7 .72966 | 1.37050 | .75675 | 1.32144 | .78457 | 1.27458 | .81316 | 1.22977 |
| 8 .73010 | 1.36967 | .75721 | 1.32064 | .78504 | 1.27382 | .81364 | 1.22904 |
| 9 .73055 | 1.36883 | .75767 | 1.31984 | .78551 | 1.27306 | .81413 | 1.22831 |
| 10 .73100 | 1.36800 | .75812 | 1.31904 | .78598 | 1.27230 | .81461 | 1.22758 |
| 11 .73144 | 1.36716 | .75858 | 1.31825 | .78645 | 1.27153 | .81510 | 1.22685 |
| 12 .73189 | 1.36633 | .75904 | 1.31745 | .78692 | 1.27077 | .81558 | 1.22618 |
| 13 .73234 | 1.36549 | .75950 | 1.31666 | .78739 | 1.27001 | .81606 | 1.22539 |
| 14 .73278 | 1.36466 | .75996 | 1.31586 | .78786 | 1.26925 | .81655 | 1.22467 |
| 15 .73323 | 1.36383 | .76042 | 1.31507 | .78834 | 1.26849 | .81703 | 1.22394 |
| 16 .73368 | 1.36300 | .76088 | 1.31427 | .78881 | 1.26774 | .81752 | 1.22321 |
| 17 .73413 | 1.36217 | .76134 | 1.31348 | .78928 | 1.26698 | .81800 | 1.22249 |
| 18 .73457 | 1.36134 | .76180 | 1.31269 | .78975 | 1.26622 | .81849 | 1.22176 |
| 19 .73502 | 1.36051 | .76226 | 1.31190 | .79022 | 1.26546 | .81898 | 1.22104 |
| 20 .73547 | 1.35968 | .76272 | 1.31110 | .79070 | 1.26471 | .81946 | 1.22031 |
| 21 .73592 | 1.35885 | .76318 | 1.31031 | .79117 | 1.26395 | .81995 | 1.21959 |
| 22 .73637 | 1.35802 | .76364 | 1.30952 | .79164 | 1.26319 | .82044 | 1.21886 |
| 23 .73681 | 1.35719 | .76410 | 1.30873 | .79212 | 1.26244 | .82092 | 1.21814 |
| 24 .73726 | 1.35637 | .76456 | 1.30795 | .79259 | 1.26169 | .82141 | 1.21742 |
| 25 .73771 | 1.35554 | .76502 | 1.30716 | .79306 | 1.26093 | .82190 | 1.21670 |
| 26 .73816 | 1.35472 | .76548 | 1.30637 | .79354 | 1.26018 | .82238 | 1.21598 |
| 27 .73861 | 1.35389 | .76594 | 1.30558 | .79401 | 1.25943 | .82287 | 1.21526 |
| 28 .73906 | 1.35307 | .76640 | 1.30480 | .79449 | 1.25867 | .82336 | 1.21454 |
| 29 .73951 | 1.35224 | .76686 | 1.30401 | .79496 | 1.25792 | .82385 | 1.21388 |
| 30 .73996 | 1.35142 | .76732 | 1.30323 | .79544 | 1.25717 | .82434 | 1.21310 |
| 31 .74041 | 1.35060 | .76779 | 1.30244 | .79691 | 1.25642 | .82483 | 1.21238 |
| 32 .74086 | 1.34978 | .76825 | 1.30165 | .79639 | 1.25567 | .82531 | 1.21166 |
| 33 .74131 | 1.34895 | .76871 | 1.30087 | .79686 | 1.25492 | .82580 | 1.21094 |
| 34 .74176 | 1.34814 | .76918 | 1.30009 | .79734 | 1.25417 | .82629 | 1.21023 |
| 35 .74221 | 1.34732 | .76964 | 1.29931 | .79781 | 1.25343 | .82678 | 1.20951 |
| 36 .74267 | 1.34650 | .77010 | 1.29853 | .79829 | 1.25268 | .82727 | 1.20879 |
| 37 .74312 | 1.34568 | .77057 | 1.29775 | .79877 | 1.25198 | .82776 | 1.20808 |
| 38 .74357 | 1.34487 | .77103 | 1.29696 | .79924 | 1.25118 | .82825 | 1.20736 |
| 39 .74403 | 1.34405 | .77149 | 1.29618 | .79973 | 1.25044 | .82874 | 1.20665 |
| 40 .74447 | 1.34323 | .77196 | 1.29541 | .80020 | 1.24989 | .82923 | 1.20598 |
| 41 .74498 | 1.34242 | .77243 | 1.29463 | .80067 | 1.24905 | .82972 | 1.20522 |
| 42 .74538 | 1.34160 | .77289 | 1.29385 | .80115 | 1.24830 | .83022 | 1.20451 |
| 43 .74588 | 1.34079 | .77335 | 1.29307 | .80163 | 1.24746 | .83071 | 1.20379 |
| 44 .74628 | 1.33998 | .77382 | 1.29229 | .80211 | 1.24673 | .83120 | 1.20308 |
| 45 .74674 | 1.33916 | .77428 | 1.29152 | .80258 | 1.24597 | .83169 | 1.20237 |
| 46 .74719 | 1.33835 | .77475 | 1.29074 | .80306 | 1.24523 | .83218 | 1.20166 |
| 47 .74764 | 1.33754 | .77521 | 1.28997 | .80354 | 1.24449 | .83268 | 1.20094 |
| 48 .74810 | 1.33673 | .77568 | 1.28919 | .80402 | 1.24375 | .83317 | 1.20024 |
| 49 .74855 | 1.33592 | .77615 | 1.28842 | .80450 | 1.24301 | .83366 | 1.19953 |
| 50 .74900 | 1.33511 | .77661 | 1.28764 | .80498 | 1.24227 | .83415 | 1.19883 |
| 51 .74946 | 1.33430 | .77708 | 1.28687 | .80546 | 1.24153 | .83465 | 1.19811 |
| 52 .74991 | 1.33349 | .77754 | 1.28610 | .80594 | 1.24079 | .83514 | 1.19740 |
| 53 .75037 | 1.33268 | .77801 | 1.28533 | .80642 | 1.24005 | .83564 | 1.19669 |
| 54 .75083 | 1.33187 | .77848 | 1.28456 | .80690 | 1.23931 | .83613 | 1.19599 |
| 55 .75128 | 1.33107 | .77895 | 1.28379 | .80738 | 1.23856 | .83662 | 1.19528 |
| 56 .75173 | 1.33026 | .77941 | 1.28302 | .80786 | 1.23784 | .83712 | 1.19457 |
| 57 .75219 | 1.32946 | .77988 | 1.28225 | .80834 | 1.23710 | .83761 | 1.19387 |
| 58 .75264 | 1.32865 | .78035 | 1.28148 | .80883 | 1.23637 | .83811 | 1.19316 |
| 59 .75310 | 1.32785 | .78082 | 1.28071 | .80930 | 1.23563 | .83860 | 1.19246 |
| 60 .75355 | 1.32704 | .78129 | 1.27994 | .80978 | 1.23490 | .83910 | 1.19175 |
| Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |
| 53° | 52° | 51° | 50° | | | | |

NATURAL TANGENTS AND COTANGENTS. 135

| | 40° | | 41° | | 42° | | 43° | | |
|----|--------|---------|--------|---------|--------|---------|--------|---------|----|
| | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | |
| 0 | .89910 | 1.19175 | .86929 | 1.15087 | .90040 | 1.11061 | .93252 | 1.07287 | 60 |
| 1 | .89960 | 1.19105 | .86980 | 1.14969 | .90093 | 1.10996 | .93306 | 1.07174 | 59 |
| 2 | .84009 | 1.19035 | .87031 | 1.14902 | .90146 | 1.10931 | .93360 | 1.07112 | 58 |
| 3 | .84059 | 1.18964 | .87082 | 1.14854 | .90199 | 1.10867 | .93415 | 1.07049 | 57 |
| 4 | .84108 | 1.18894 | .87133 | 1.14767 | .90251 | 1.10802 | .93469 | 1.06987 | 56 |
| 5 | .84158 | 1.18824 | .87184 | 1.14699 | .90304 | 1.10737 | .93524 | 1.06925 | 55 |
| 6 | .84208 | 1.18754 | .87236 | 1.14633 | .90357 | 1.10673 | .93578 | 1.06863 | 54 |
| 7 | .84258 | 1.18684 | .87287 | 1.14565 | .90410 | 1.10607 | .93633 | 1.06800 | 53 |
| 8 | .84307 | 1.18614 | .87338 | 1.14498 | .90463 | 1.10543 | .93688 | 1.06738 | 52 |
| 9 | .84357 | 1.18544 | .87389 | 1.14430 | .90516 | 1.10478 | .93742 | 1.06676 | 51 |
| 10 | .84407 | 1.18474 | .87441 | 1.14363 | .90569 | 1.10414 | .93797 | 1.06613 | 50 |
| 11 | .84457 | 1.18404 | .87492 | 1.14296 | .90621 | 1.10349 | .93852 | 1.06551 | 49 |
| 12 | .84507 | 1.18334 | .87543 | 1.14229 | .90674 | 1.10285 | .93906 | 1.06489 | 48 |
| 13 | .84556 | 1.18264 | .87595 | 1.14163 | .90727 | 1.10220 | .93961 | 1.06427 | 47 |
| 14 | .84606 | 1.18194 | .87646 | 1.14095 | .90781 | 1.10156 | .94016 | 1.06365 | 46 |
| 15 | .84656 | 1.18125 | .87698 | 1.14028 | .90834 | 1.10091 | .94071 | 1.06308 | 45 |
| 16 | .84706 | 1.18055 | .87749 | 1.13961 | .90887 | 1.10027 | .94125 | 1.06241 | 44 |
| 17 | .84756 | 1.17986 | .87801 | 1.13894 | .90940 | 1.09963 | .94180 | 1.06179 | 43 |
| 18 | .84806 | 1.17916 | .87852 | 1.13828 | .90993 | 1.09899 | .94235 | 1.06117 | 42 |
| 19 | .84856 | 1.17846 | .87904 | 1.13761 | .91046 | 1.09834 | .94290 | 1.06056 | 41 |
| 20 | .84906 | 1.17777 | .87955 | 1.13694 | .91099 | 1.09770 | .94345 | 1.05994 | 40 |
| 21 | .84956 | 1.17708 | .88007 | 1.13627 | .91153 | 1.09706 | .94400 | 1.05932 | 39 |
| 22 | .85006 | 1.17638 | .88059 | 1.13561 | .91206 | 1.09642 | .94455 | 1.05870 | 38 |
| 23 | .85057 | 1.17569 | .88110 | 1.13494 | .91259 | 1.09578 | .94510 | 1.05809 | 37 |
| 24 | .85107 | 1.17500 | .88162 | 1.13428 | .91313 | 1.09514 | .94565 | 1.05747 | 36 |
| 25 | .85157 | 1.17430 | .88214 | 1.13361 | .91366 | 1.09450 | .94620 | 1.05685 | 35 |
| 26 | .85207 | 1.17361 | .88265 | 1.13295 | .91419 | 1.09386 | .94676 | 1.05624 | 34 |
| 27 | .85257 | 1.17292 | .88317 | 1.13228 | .91473 | 1.09322 | .94731 | 1.05562 | 33 |
| 28 | .85308 | 1.17223 | .88369 | 1.13162 | .91526 | 1.09258 | .94786 | 1.05501 | 32 |
| 29 | .85358 | 1.17154 | .88421 | 1.13096 | .91580 | 1.09195 | .94841 | 1.05439 | 31 |
| 30 | .85408 | 1.17084 | .88473 | 1.13029 | .91633 | 1.09131 | .94896 | 1.05378 | 30 |
| 31 | .85458 | 1.17016 | .88524 | 1.12963 | .91687 | 1.09067 | .94952 | 1.05317 | 29 |
| 32 | .85509 | 1.16947 | .88576 | 1.12897 | .91740 | 1.09003 | .95007 | 1.05255 | 28 |
| 33 | .85559 | 1.16878 | .88628 | 1.12831 | .91794 | 1.08940 | .95062 | 1.05194 | 27 |
| 34 | .85609 | 1.16809 | .88680 | 1.12765 | .91847 | 1.08876 | .95118 | 1.05138 | 26 |
| 35 | .85660 | 1.16741 | .88732 | 1.12699 | .91901 | 1.08818 | .95173 | 1.05073 | 25 |
| 36 | .85710 | 1.16672 | .88784 | 1.12633 | .91955 | 1.08749 | .95229 | 1.05010 | 24 |
| 37 | .85761 | 1.16603 | .88836 | 1.12567 | .92008 | 1.08684 | .95284 | 1.04949 | 23 |
| 38 | .85811 | 1.16535 | .88888 | 1.12501 | .92062 | 1.08622 | .95340 | 1.04888 | 22 |
| 39 | .85862 | 1.16466 | .88940 | 1.12435 | .92116 | 1.08559 | .95395 | 1.04827 | 21 |
| 40 | .85912 | 1.16398 | .88992 | 1.12369 | .92170 | 1.08496 | .95451 | 1.04766 | 20 |
| 41 | .86963 | 1.16329 | .89045 | 1.12303 | .92224 | 1.08432 | .96506 | 1.04705 | 19 |
| 42 | .86014 | 1.16261 | .89097 | 1.12238 | .92277 | 1.08369 | .95562 | 1.04644 | 18 |
| 43 | .86064 | 1.16192 | .89149 | 1.12172 | .92331 | 1.08306 | .95618 | 1.04588 | 17 |
| 44 | .86115 | 1.16124 | .89201 | 1.12106 | .92385 | 1.08243 | .95673 | 1.04522 | 16 |
| 45 | .86166 | 1.16056 | .89253 | 1.12041 | .92439 | 1.08179 | .95729 | 1.04461 | 15 |
| 46 | .86216 | 1.15987 | .89306 | 1.11975 | .92493 | 1.08116 | .95785 | 1.04401 | 14 |
| 47 | .86267 | 1.15919 | .89358 | 1.11909 | .92547 | 1.08053 | .95841 | 1.04340 | 13 |
| 48 | .86318 | 1.15851 | .89410 | 1.11844 | .92601 | 1.07990 | .95897 | 1.04279 | 12 |
| 49 | .86368 | 1.15783 | .89463 | 1.11778 | .92655 | 1.07927 | .96052 | 1.04218 | 11 |
| 50 | .86419 | 1.15715 | .89515 | 1.11713 | .92709 | 1.07864 | .96008 | 1.04158 | 10 |
| 51 | .86470 | 1.15647 | .89567 | 1.11648 | .92763 | 1.07801 | .96064 | 1.04097 | 9 |
| 52 | .86521 | 1.15579 | .89620 | 1.11582 | .92817 | 1.07738 | .96120 | 1.04036 | 8 |
| 53 | .86572 | 1.15511 | .89672 | 1.11517 | .92872 | 1.07676 | .96176 | 1.03976 | 7 |
| 54 | .86623 | 1.15443 | .89725 | 1.11452 | .92926 | 1.07613 | .96238 | 1.03915 | 6 |
| 55 | .86674 | 1.15375 | .89777 | 1.11387 | .92980 | 1.07550 | .96288 | 1.03855 | 5 |
| 56 | .86725 | 1.15308 | .89830 | 1.11321 | .93034 | 1.07487 | .96344 | 1.03794 | 4 |
| 57 | .86776 | 1.15240 | .89883 | 1.11256 | .93086 | 1.07425 | .96400 | 1.03734 | 3 |
| 58 | .86827 | 1.15172 | .89935 | 1.11191 | .93143 | 1.07362 | .96457 | 1.03674 | 2 |
| 59 | .86878 | 1.15104 | .89988 | 1.11126 | .93197 | 1.07299 | .96513 | 1.03613 | 1 |
| 60 | .86929 | 1.15037 | .90040 | 1.11061 | .93252 | 1.07237 | .96569 | 1.03558 | 0 |
| | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | |
| | 49° | | 48° | | 47° | | 46° | | |

136 NATURAL TANGENTS AND COTANGENTS.

| 44° | | | | 44° | | | | 44° | | | | | | |
|-----|--------|---------|----|-----|--------|---------|----|-----|---------|---------|-----|--|--|--|
| | Tang | Cotang | | | Tang | Cotang | | | Tang | Cotang | | | | |
| 0 | .96569 | 1.08553 | 60 | 20 | .97700 | 1.02355 | 40 | 40 | .99843 | 1.01170 | 20 | | | |
| 1 | .96625 | 1.08498 | 59 | 21 | .97756 | 1.02295 | 39 | 41 | .99801 | 1.01113 | 19 | | | |
| 2 | .96681 | 1.08433 | 58 | 22 | .97813 | 1.02236 | 38 | 42 | .99768 | 1.01053 | 18 | | | |
| 3 | .96738 | 1.08373 | 57 | 23 | .97870 | 1.02176 | 37 | 43 | .99716 | 1.00994 | 17 | | | |
| 4 | .96794 | 1.08312 | 56 | 24 | .97927 | 1.02117 | 36 | 44 | .99673 | 1.00935 | 16 | | | |
| 5 | .96850 | 1.08253 | 55 | 25 | .97984 | 1.02057 | 35 | 45 | .99131 | 1.00876 | 15 | | | |
| 6 | .96907 | 1.08192 | 54 | 26 | .98041 | 1.01998 | 34 | 46 | .99189 | 1.00818 | 14 | | | |
| 7 | .96963 | 1.08135 | 53 | 27 | .98098 | 1.01938 | 33 | 47 | .99247 | 1.00759 | 13 | | | |
| 8 | .97020 | 1.08072 | 52 | 28 | .98155 | 1.01879 | 32 | 48 | .99304 | 1.00701 | 12 | | | |
| 9 | .97076 | 1.08012 | 51 | 29 | .98213 | 1.01820 | 31 | 49 | .99362 | 1.00642 | 11 | | | |
| 10 | .97133 | 1.02953 | 50 | 30 | .98270 | 1.01761 | 30 | 50 | .99420 | 1.00583 | 10 | | | |
| 11 | .97189 | 1.02892 | 49 | 31 | .98327 | 1.01702 | 29 | 51 | .99478 | 1.00525 | 9 | | | |
| 12 | .97246 | 1.02833 | 48 | 32 | .98384 | 1.01643 | 28 | 52 | .99536 | 1.00467 | 8 | | | |
| 13 | .97303 | 1.02772 | 47 | 33 | .98441 | 1.01583 | 27 | 53 | .99594 | 1.00408 | 7 | | | |
| 14 | .97359 | 1.02713 | 46 | 34 | .98499 | 1.01524 | 26 | 54 | .99652 | 1.00350 | 6 | | | |
| 15 | .97416 | 1.02653 | 45 | 35 | .98556 | 1.01465 | 25 | 55 | .99710 | 1.00291 | 5 | | | |
| 16 | .97472 | 1.02593 | 44 | 36 | .98613 | 1.01406 | 24 | 56 | .99768 | 1.00233 | 4 | | | |
| 17 | .97529 | 1.02533 | 43 | 37 | .98671 | 1.01347 | 23 | 57 | .99826 | 1.00175 | 3 | | | |
| 18 | .97586 | 1.02474 | 42 | 38 | .98728 | 1.01288 | 22 | 58 | .99884 | 1.00116 | 2 | | | |
| 19 | .97643 | 1.02414 | 41 | 39 | .98786 | 1.01229 | 21 | 59 | .99942 | 1.00058 | 1 | | | |
| 20 | .97700 | 1.02355 | 40 | 40 | .98843 | 1.01170 | 20 | 60 | 1.00000 | 1.00000 | 0 | | | |
| | Cotang | Tang | | | Cotang | Tang | | | Cotang | Tang | | | | |
| | 45° | | | | | 45° | | | | | 45° | | | |

APPENDIX A.

REINFORCED-CONCRETE RETAINING-WALLS.

A RETAINING-WALL constructed of reinforced concrete may be considered as a vertical beam acting as a cantilever securely anchored to a horizontal beam which also may be considered as cantilevered at each end. This becomes clear upon considering Fig. 1. The wall *ABGF* is a cantilevered beam anchored at *FG*, where the section must be sufficient to safely resist the horizontal thrust of the earth above the plane of *FG* and the moment produced by this thrust. The vertical weight of the wall also produces compression at the section, but, as will appear later, this is comparatively small and may be neglected. The same conditions obtain for each section of the wall above *FG*.

Considering the wall and foundation as a whole, the intensities along *LK* have a tendency to bend upward the cantilever *MF*; while the weight of the earth above *GI* combined with the upward intensities along *KL* has a tendency to rupture the cantilever *GI* at *G*.

In order to design a wall of this type it will be necessary to state the formulas upon which the design of reinforced beams is based.

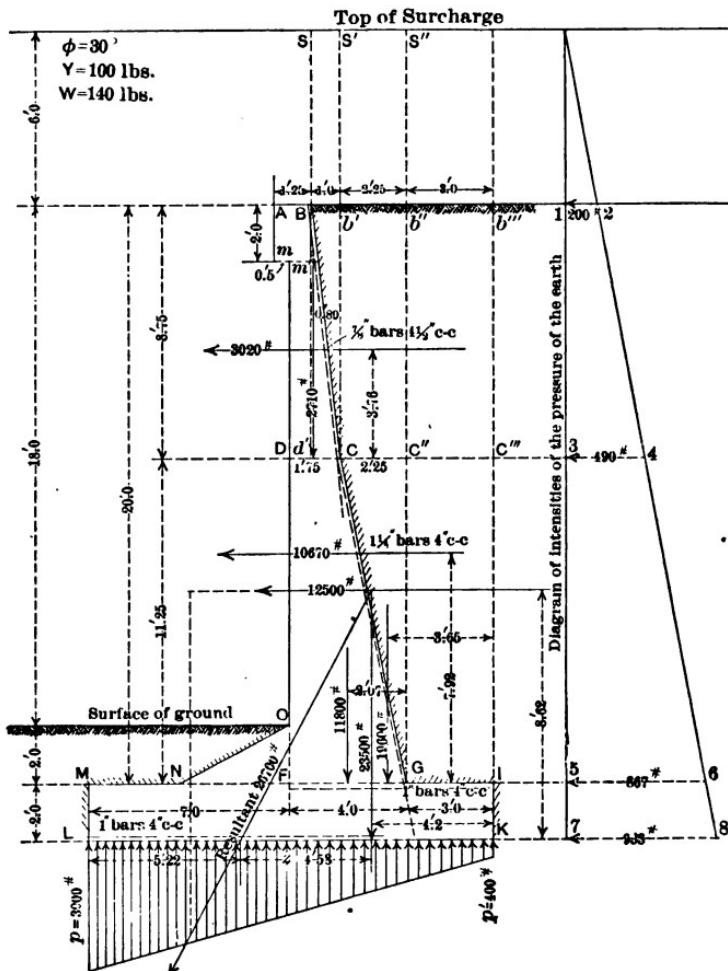


FIG. 1.

The usual reinforcement is in the form of plain or deformed steel rods.

The formulas used will be those given in the Standard Specifications adopted August 15, 1908, by the American Society for Testing Materials.

NOMENCLATURE.

(a) Rectangular Beams.

f_s = tensile unit stress in steel;

f_c = compressive unit stress in concrete;

E_s = modulus of elasticity of steel;

E_c = modulus of elasticity of concrete;

$n = E_s \div E_c$;

M = moment of resistance, or bending moment in general;

A = area of steel;

b = breadth of beam;

d = depth of beam to center of steel;

k = ratio of depth of neutral axis to effective depth d ;

z = depth of resultant compression below top;

j = ratio of lever arm of resisting couple to depth d ;

$jd = d - z$ = arm of resisting couple;

p = steel ratio = $A \div bd$.

Rectangular Beams Reinforced for Compression.

A' = area of compressive steel;

p' = steel ratio = $A' \div bd$;

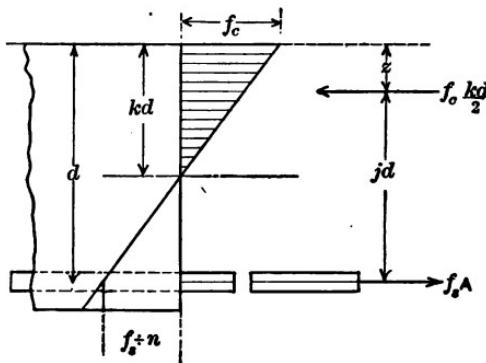
f'_s = unit compressive stress in steel;

C = total compressive stress in concrete;

$C' =$ total compressive stress in steel = $f'_s A'$;

d' = depth to center of compressive steel;

z = depth to resultant of C and C' .

Shear and Bond. V = total shear; v = shearing unit stress (lengthwise of beam just above bars); u = bond stress per unit area of bar (resisted by adhesion between cement and steel); o = circumference or perimeter of bar; Σo = sum of the perimeters of all bars.(b). *Formulas.*(1) *Rectangular Beams.*

$$k = \sqrt{2pn + (pn)^2} - pn, \quad \dots \quad (1)$$

$$j = 1 - \frac{1}{3}k, \quad \dots \quad (2)$$

$$f_s = \frac{M}{Ajd} = \frac{M}{pjbd^2}, \quad \dots \quad (3)$$

$$M = pjf_s bd^2, \quad \dots \quad (4)$$

$$f_c = \frac{2M}{jkb d^2} = \frac{2pf_s}{k}, \quad \dots \quad (5)$$

$$M = \frac{1}{2} j k f_c b d^2, \dots \dots \dots \quad (6)$$

$$p = \frac{1}{2} \frac{1}{f_c \left(\frac{f_s}{n f_c} + 1 \right)}. \dots \dots \dots \quad (7)$$

For $f_s = 15,000$ to 16,000 and $f_c = 600$ to 650, j may be taken at $\frac{7}{8}$ and hence $k = \frac{3}{8}$.

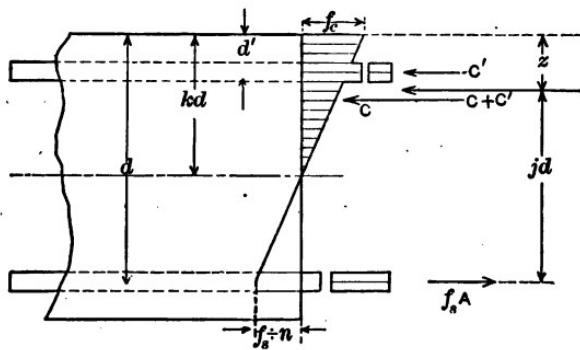
Then

$$M = 0.865 A f_s d, \dots \dots \dots \quad (8)$$

and

$$M = 0.164 f_c b d^2. \dots \dots \dots \quad (9)$$

(2) Rectangular Beams Reinforced for Compression.



$$k = \sqrt{2n \left(p + p' \frac{d'}{d} \right) + n^2(p+p')^2 - n(p+p')}, \dots \quad (10)$$

$$z = \frac{\frac{1}{3} k^3 d + 2p' n d' \left(k - \frac{d'}{d} \right)}{k^2 + 2p' n \left(k - \frac{d'}{d} \right)} \dots \dots \dots \quad (11)$$

$$jd = d - z, \dots \dots \dots \dots \dots \dots \dots \quad (12)$$

$$f_c = \frac{6M}{bd^2 \left\{ 3k - k^2 + \frac{6p'n}{k} \left(k - \frac{d'}{d} \right) \left(1 - \frac{d'}{d} \right) \right\}}. \quad (13)$$

$$f_s = \frac{M}{pjbd^2} = nf_c \frac{1-k}{k}. \quad \dots \dots \dots \dots \dots \quad (14)$$

$$f'_s = nf_c \frac{k - \frac{d'}{d}}{k}, \quad \dots \dots \dots \dots \dots \quad (15)$$

(3) *Shear, Bond, and Web Reinforcement.*

For rectangular beams,

$$v = \frac{V}{bjd}, \quad \dots \dots \dots \dots \dots \quad (16)$$

$$u = \frac{V}{jd\Sigma o}, \quad \dots \dots \dots \dots \dots \quad (17)$$

For web reinforcement,

$$P = \frac{V's}{jd} \text{ (vertical reinforcement)}, \quad \dots \dots \quad (18)$$

$$P = 0.7 \frac{V's}{jd} \text{ (diagonal reinforcement)}, \quad \dots \quad (19)$$

in which P = stress in horizontal reinforcement, V' = portion of total vertical shear assumed as carried by the web

reinforcement, and s = horizontal spacing of the reinforcing bars.

Approximately, jd may be taken as $\frac{1}{8}$ for the usual working stresses. Then

$$v = \frac{8V}{7b} \quad \text{and} \quad u = \frac{8V}{7\sum o},$$

also

$$P = \frac{8}{7}V's \quad \text{and} \quad P = \frac{8}{10}V's.$$

The formulas given above are based on the following assumptions:

- (1) A plane section before bending remains plane after bending.
- (2) The modulus of elasticity of concrete in compression, within the usual limits of working stresses, is constant. The distribution of compressive stresses in beams is therefore rectilinear.
- (3) In calculating the moment of resistance of beams the tensile stresses in the concrete shall be neglected.
- (4) Perfect adhesion is assumed between concrete and reinforcement. Under compressive stresses the two materials are therefore stressed in proportion to their moduli of elasticity.
- (5) Initial stress in the reinforcement due to contraction or expansion in the concrete is neglected.

The following working stresses will be used for concrete having an ultimate compressive strength of 2000 pounds at the end of 28 days.

| | |
|---|----------------------|
| Compression in extreme fiber | 650 lbs. per sq. in. |
| Tension in extreme fiber | 0 " |
| Shear, pure | 120 " |
| Shear, combined with other stresses . . | 40 " |
| Bond between concrete and steel | 80 " |
| Bond between concrete and drawn wire | 40 " |

Steel Reinforcement. Tensile stress not to exceed 16,000 pounds per square inch. Compressive stress not to exceed 15 times the working compressive stress in the concrete.

$$E_s \div E_c = 15.$$

In order that the compressive stress in the concrete shall be 650 pounds per square inch and the tensile stress in the steel 16,000 pounds per square inch, p must equal about 0.0077. Then $k=0.378$ and $j=0.873$.

From either (4) or (6) (nearly)

$$M = 107.6bd^2 \quad \text{or} \quad 108d^2 \text{ say, for } b=\text{unity.}$$

Then

$$d = \sqrt{\frac{M}{108}} \quad \dots \dots \dots \dots \dots \quad (A)$$

Since $p=0.0077$,

$$A = 0.0077d \text{ for } b=\text{unity.} \quad \dots \dots \dots \quad (B)$$

The following deformed bars are in general use at the present time. *Small bars spaced at short intervals are preferable to large bars at long intervals.*

AREAS AND WEIGHTS OF DEFORMED BARS.
(From Catalogues.)

| Size, Inches. | Johnson Old Style. | | Johnson New Style. | | Johnson Universal. | | Thacher Patent. | | Ransome Twisted. | | Plain Round. | |
|------------------|-----------------------|------|-----------------------|------|-----------------------|------|--------------------|------|---------------------|------|-----------------|------|
| | *A | *W | A | W | A | W | A | W | A | W | A | W |
| 1 | 0.18 | 0.64 | 0.25 | 0.85 | 0.32a | 1.18 | 0.18 | 0.61 | 0.25 | 0.85 | 0.20 | 0.67 |
| | 0.37 | 1.35 | 0.56 | 1.91 | 0.41b | 1.35 | 0.41 | 1.39 | 0.56 | 1.91 | 0.44 | 1.50 |
| $\frac{7}{8}$ | 0.55 | 1.95 | 0.77 | 2.60 | 0.54c | 1.97 | 0.55 | 1.87 | 0.77 | 2.60 | 0.60 | 2.04 |
| 1 | 0.70 | 2.70 | 1.00 | 3.40 | 0.65d | 2.27 | 0.71 | 2.42 | 1.00 | 3.40 | 0.79 | 2.67 |
| $1\frac{1}{4}$ | 1.07 | 4.00 | 1.56 | 5.31 | 0.80e | 2.85 | 1.10 | 3.74 | 1.56 | 5.31 | 1.23 | 4.17 |

* A = net area in square inches; W = weight in pounds per linear foot.
a, $\frac{1}{8}'' \times 1\frac{1}{4}$; b, $\frac{1}{8}'' \times 1\frac{1}{2}$; c, $\frac{1}{8}'' \times 1\frac{1}{4}$; d, $\frac{1}{8}'' \times 2''$; e, $\frac{1}{8}'' \times 2\frac{1}{4}$.

EXAMPLE 1 (Fig. 1, page 138). Investigate the strength and stability of the wall and foundation *ABKL*, Fig. 1. Wall and foundation composed of ordinary stone portland concrete proportioned 1:3:6 and weighing 140 pounds per cubic foot. The reinforcement is of medium steel having an ultimate strength of 64,000 pounds per square inch and a modulus of elasticity of 30,000,000. The earth weighs 100 pounds per cubic foot and has an angle of repose of 30° . The surcharge load is equivalent to 6 feet of earth.

SECTION DC.

The intensity of the earth-pressure normal to a vertical plane at any depth *H* is given by the formula (page 38)

$$q = H\gamma \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{100}{3}H;$$

hence the intensity at b' is 200 pounds and at C 490 pounds. The total thrust against the plane $b'C$ equals $\frac{200+490}{2} \times 8.75$ or 3020 pounds. The application of this thrust is 3.76 feet above DC , or through the center of gravity of the intensity area 1243.

The center of gravity of the wall above DC and the earth prism $SS'b'B$ can be found as follows:

Moments about $b'C$:

$$\begin{array}{rcl}
 SS'b'B \dots & [1.0 \times 6.0 \times 100 = 600] 0.5 & = 300 \\
 Bb'C \dots \dots & \frac{1}{2}[1.0 \times 8.75 \times 100 = 438] \frac{1}{2} & = 146 \\
 & \hline & \\
 & [1038] 0.43 & = 446 \\
 ABm'm \dots & [1.25 \times 2 \times 140 = 350] 1.625 & = 568.8 \\
 mm'Dd' \dots & [0.75 \times 6.75 \times 140 = 709] 1.375 & = 974.9 \\
 BCd' \dots \dots & \frac{1}{2}[1.0 \times 8.75 \times 140 = 613] 0.667 & = 408.7 \\
 & \hline & \\
 & [1672] 1.17 & = 1952.4
 \end{array}$$

The center of gravity of the earth prism is 0.43 of a foot from $b'C$ and that of the wall 1.17 feet.

$$\begin{array}{rcl}
 \text{Earth.} \dots \dots \dots & [1038] 0.43 & = 446 \\
 \text{Wall.} \dots \dots \dots & [1672] 1.17 & = 1952 \\
 & \hline & \\
 \text{Combination.} \dots \dots & [2710] 0.89 & = 2398
 \end{array}$$

The center of gravity of the combination is 0.89 of a foot from $b'C$ or 10.3 inches from D .

Assuming the steel to be placed 3 inches from the surface of the concrete, the effective depth d of the section DC is 18 inches. Seven-eighths-inch O. S. Johnson bars spaced $4\frac{1}{2}$ inches center to center are equivalent to $0.55 \div 4.5 = 0.122$ square inches per linear

inch of wall; hence $p = 0.122 \div 18 = 0.0058$. Remembering that $n = 15$ in (1), $k = 0.34$ and $kd = 6.12$ inches, the distance of the neutral axis from D .

The total moment per linear foot of wall is:

For the earth thrust, $+3020 \times 3.76 = +11400$ ft.-lbs.

For the vertical load, $-2710 \times 0.35 = -949$ " "

Total. 10451 ft.-lbs.

$10451 \div 12 = 871$ ft.-lbs. per linear inch of wall or 10451 in.-lbs. per linear inch of wall.

From (2) $j = 0.887$ and hence $jd = 16$ inches about.

From (3) $f_s = 10451 \div 1.67 = 6300$ pounds per square inch.

From (5) $f_c = 2 \times 10451 \div 98 = 213$ pounds per square inch. These values are very much smaller than those allowable, hence the section is amply safe. The effect of the direct stress due to the weight of the wall above the section is so small that it has been neglected.

SECTION FG.

The block of concrete OFN and the earth in front of the wall will be neglected. If the section FG is sufficiently strong, all sections between FG and CD will be safe.

The earth-pressure against the plane $b''G$ is $\frac{200+867}{2} 20 = 10670$ pounds per linear foot of wall, which acts 7.92 feet above FG . Taking the section as it is shown in Fig. 1, the vertical weight above FG and its point of application with reference to G can be found as follows:

| | | | |
|--------------------|-------------------------|--------------------------|-------|
| [Weight on DC | = | 2710]3.14 = | 8509 |
| S'S''C''C. | [2.25 × 14.75 × 100 = | 3320] $\frac{2.25}{2}$ = | 3735 |
| CC''G. | { [2.25 × 11.25 × 100 = | 1265] $\frac{2.25}{3}$ = | 948 |
| FDC. | { [1.75 × 11.25 × 140 = | 2771]3.12 = | 8646 |
| | { [2.25 × 11.25 × 140 = | 1771]1.50 = | 2656 |
| | | | <hr/> |
| | | [11837]2.07 = | 24494 |

The total vertical load is 11,800 pounds per linear foot of wall and acts 2.07 feet from G'. The effective depth for FG is $48 - 3 = 45$ inches; $1\frac{1}{4}$ " O. S. Johnson bars spaced 4 inches center to center are equivalent to $1.07 \div 4 = 0.2675$ square inches per linear inch of wall; hence $p = 0.2675 \div 45 = 0.006$ nearly.

From (1) $k = 0.353$ and $kd = 15.9$ inches. The distance of the neutral axis from F,

From (2) $j = 0.882$.

The total moment per linear foot of wall is:

For the earth thrust, $+10670 \times 7.92 = +84500$ ft.-lbs.

For the vertical load, $-11800 \times 0.47 = -5550$ "

Total 78950

or $78950 \div 12 = 6580$ ft.-lbs. per linear inch of wall or 78950 in.-lbs. per linear inch of wall.

From (3) $f_s = 78950 \div 107.2 = 7360$ pounds per square inch.

From (5) $f_c = 2 \times 78950 \div 630 = 260$ pounds per square inch. The values of f_s and f_c are much smaller than those allowed.

A review of the above calculations shows that the effect of the vertical load and its moment is too small for consideration, and consequently the sections of the wall can usually be designed for the *moment produced by the earth-pressure alone*. The earth-pressure may be assumed to act against a vertical plane and, hence, horizontally, unless the back of the wall has a great inclination from the vertical.

Foundation.

The earth-pressure against the vertical plane Kb'' is 12,500 pounds per foot of wall and acts 8.62 ft. above LK . The earth in front of the wall will be neglected. The total vertical pressure on the plane LK is 23,500 pounds per foot of wall and acts 4.2 feet from K .

Taking moments about the point where the resultant of the earth-pressure and the vertical load cuts LK ,

$$23500 \times z = 12500 \times 8.62. \quad \therefore z = 4.58 \text{ ft.}$$

The resultant, then, cuts LK 5.22 ft. from L and $7.00 - 5.22 = 1.78$ ft. from the centre of LK ; hence $x_0 = 1.78$ ft.

The intensity of the vertical pressure at the toe of the foundation is (page 31)

$$p = \left\{ 1 + \frac{6x_0}{B} \right\} p_0 = \left\{ 1 + \frac{6 \times 1.78}{14} \right\} \frac{23500}{14} = 3900 \text{ lbs. per sq. ft.}$$

Since $p' = 2p_0 - p$, $p' = 400$ lbs. per square foot. The allowable intensity at the toe is (eq. 17, page 39)

$$p = 4 \times 100 \{ 9 \} = 3600 \text{ lbs. per sq. ft.},$$

and the least allowable intensity at the heel is (eq. 18, page 40)

$$p' = 28 \times 100 \{ 0.11 \} = 308 \text{ lbs. per sq. ft.}$$

As the actual intensity at the toe is less than the allowable, and that at the heel greater than required, the spread and depth of the foundation is fairly satisfactory. A somewhat greater spread or a greater depth would be better.

For the first few weeks after the wall is completed, and probably even after the earth has been filled in back of the wall, the earth in front of the foundation cannot be considered as opposing the sliding of the wall along *LK*. If undisturbed it would require a horizontal force of (eq. 26, page 43)

$$P = \frac{(4)^2 100}{2} 3 = 2400 \text{ lbs. per foot of wall}$$

to overcome its resistance. This is about one-fifth the total horizontal force acting.

The resultant makes an angle with the vertical which has a tangent of $\frac{12500}{23500} = 0.532$. From Table II, page 111, it appears that the structure is just about safe against sliding forward if ϕ remains 30° .

The portion of the foundation upon the left of *F* may be considered as a cantilever acted upon by the intensities indicated in the figure. Neglecting all materials above the plane *MF*, the moment about *F* is

$$\left. \begin{array}{l} [1700 \times 7 = 11900] 3.5 = 41650 \\ [1300 \times 3.5 = 4550] 4.67 = 21250 \end{array} \right\} \text{upward};$$

$$\underline{\hspace{2cm}} \quad 62900$$

$$[2 \times 7 \times 140 = 1960] 3.5 = \underline{\hspace{2cm}} \quad 6860 \} \text{downward.}$$

$$\begin{array}{l} \text{Total moment} = 5604 \text{ ft.-lbs. per foot of wall} \\ \text{or} \qquad \qquad \qquad 4670 \text{ " " " inch " " } \end{array}$$

Placing the reinforcing steel $1\frac{1}{2}$ inches from the surface of the concrete the effective depth of the section at F

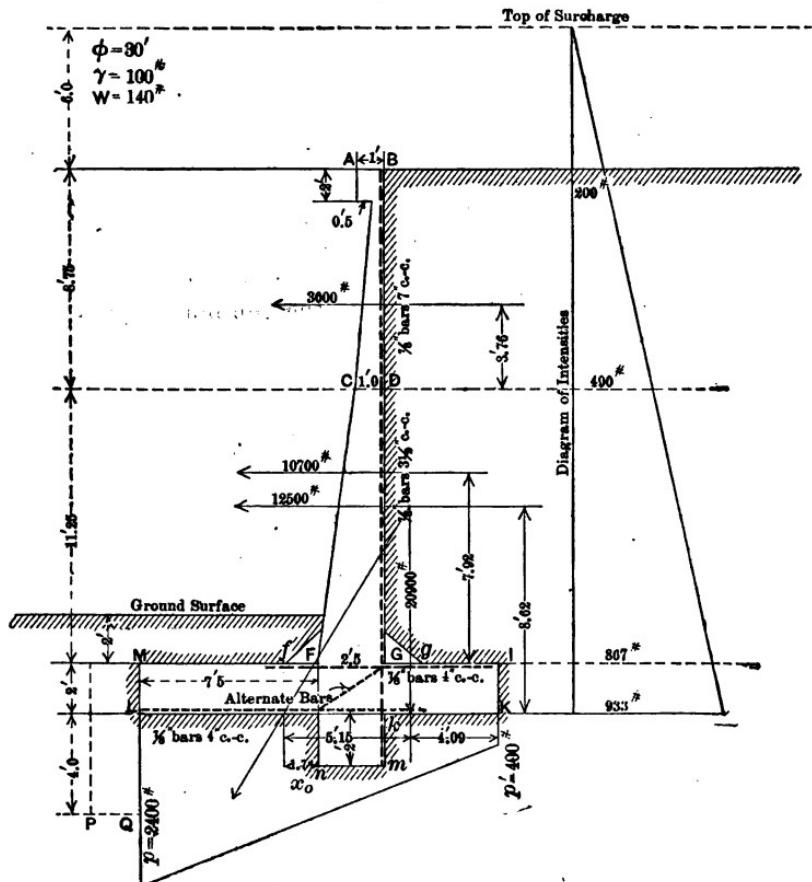


FIG. 2.

is $24 - 1.5 = 22.5$ inches. 1" O. S. Johnson bars spaced 4 ins. centre to centre are equivalent to $0.70 \div 4 = 0.175$ sq.

in. per linear in. of wall and hence $p = 0.175 \div 22.5 = 0.0078$. For this value of p and $n = 15$, $k = 0.38$ and $j = 0.873$. $f_s = 56040 \div 3.44 = 16300$ pounds per square inch. $f_c = 56040 \times 2 \div 171.8 = 668$ pounds per square inch. The values of f_s and f_c are very little different from those allowable.

The vertical shear at the section through F is

$$11900 + 4550 - 1960 = 14500 \text{ pounds,}$$

or 7250 pounds per square foot for the depth of 24 inches. This is equivalent to about 50 pounds per square inch which is 10 pounds in excess of the allowable value. Since the concrete *NFO* built in the shearing area is actually about 4 feet deep, the section is amply safe against shearing.

Inspection shows that the main wall is safe against horizontal shear, as each square foot of section can safely resist a shear of 5760 pounds.

Example 2 (Fig. 2). Design a reinforced-concrete wall to replace the design shown in Fig. 1. Let the wall be assumed vertical on the back.

SECTION DC.

The moment of the earth-pressure is:

$$3020 \times 3.76 = 11400 \text{ ft.-lbs. per linear foot of wall}$$

or $11400 \text{ in-lbs. " " inch " "}$

Assuming $p = .0077$, the effective depth is from (A)

$$d = \sqrt{\frac{M}{108}} = \sqrt{\frac{11400}{108}} = 10.2 \text{ inches.}$$

Using $\frac{7}{8}$ " N. S. Johnson bars they must be spaced not to exceed $0.77 \div 0.00785 = 9.8$ inches centre to centre.

If the bars are placed $1\frac{1}{2}$ diameters from the surface of the concrete the total depth of the section becomes $10.2 + 1.5(\frac{7}{8}) = 11.51$ inches, say 12 inches.

The actual spacing of the bars will not be settled until the spacing of the bars below has been determined.

SECTION FG.

The moment of the earth-pressure is

$$10670 \times 7.92 = 84500 \text{ ft.-lbs. per linear foot of wall}$$

or 84500 in.-lbs. " " inch " "

From (A) $d = \sqrt{\frac{84500}{108}} = 28 \text{ inches.}$

From (B) $A = 0.0077 \times 28 = 0.216$ square inches per linear inch of wall.

Using $\frac{7}{8}$ " N. S. Johnson bars they must be spaced about $0.77 \div 0.216 = 3.5$ inches centre to centre. Let these bars be spaced 3 inches centre to centre and extend alternate bars to the full height of the wall, the others extending to section CD.

The total depth of the section should be $28 + 1.5(\frac{7}{8}) = 29.3$ inches, say 30 inches.

Foundation.

Assume the foundation concrete to be 15 ft. long and 2 feet deep, with projections as shown in the figure.

The centre of gravity of the weight of the wall, founda-

tion, and the earth supported by the concrete back of the wall (neglecting that in front) is found as follows:

Moments about IK:

$$\begin{array}{l}
 \text{Foundation.} [2.0 \times 15 \times 140 = 4200] 7.50 = 31500 \\
 \text{Wall.} [* = 3743] 5.80 = 21715 \\
 \text{Earth.} [5.0 \times 26 \times 100 = 13000] 2.5 = 32500 \\
 \hline
 & [20944] 4.09 = 85715
 \end{array}$$

Hence the resultant vertical pressure upon the earth is 20,900 pounds per foot of wall and acts 4.09 feet from *IK*. If *z* is the distance from the point of application of this resultant to the point where the resultant of this force combined with the thrust of the earth cuts the base of the foundation,

$$20900z = 125.00 \times 8.62,$$

$$\text{or } z = \frac{107750}{20900} = 5.15 \text{ feet.}$$

The distance of this point from the centre of the base is

$$x_0 = (5.15 + 4.09) - 7.5 = 1.74 \text{ feet.}$$

$$* [1.0 \times 2.0 = 2.000] 0.5 = 1.00$$

$$[0.5 \times 6.75 = 3.375] 0.25 = 0.84$$

$$[0.5 \times \frac{6.75}{z} = 1.687] 0.67 = 1.13$$

$$[1.0 \times 11.25 = 11.250] 0.50 = 5.62$$

$$[1.5 \times \frac{11.25}{2} = 8.430] 1.50 = 12.65$$

$$\overline{26.742} 0.80 = \overline{21.24}$$

$$\overline{140}$$

$$\overline{3743} \text{ lbs.}$$

$$p = \left(1 + \frac{6 \times 1.74}{15}\right) \frac{20900}{15} = 2400 \text{ pounds per square foot;}$$

$$p_0 = \frac{20900}{15} = 1400 \text{ pounds per square feet;}$$

$$p' = 2p_0 - p = 2800 - 2400 = 400 \text{ pounds per square foot.}$$

From eq. 17, page 39, the allowable value of p is

$$p = 4 \times 100 \{9.0\} = 3600 \text{ pounds per square foot.}$$

From eq. 18, page 40, the minimum value of p' is

$$p' = 28 \times 100 \{0.11\} = 308 \text{ pounds per square foot.}$$

The actual values are well within the above limits.

Let the portion of the foundation in front of the vertical plane through F be assumed as a cantilevered beam acted upon by the upward intensities as indicated. Neglecting the weight of the concrete and the earth above it the moment at F is found as follows:

$$[1000 \times \frac{7.5}{2} = 3750] \cdot 7.5 = 17750$$

$$[1400 \times 7.5 = 10500] \cdot 7.5 = 39375$$

Total moment = 57125 ft.-lbs. per foot of wall
or 4760 " " inch " "

$$\text{From (A)} \quad d = \sqrt{\frac{57125}{108}} = 23 \text{ inches.}$$

From (B) $A = 23 \times 0.0077 = 0.177$ square inch per linear inch of wall. $\frac{7}{8}$ " N. S. Johnson bars must be

spaced not to exceed $0.77 \div 0.177 = 4.3$ inches centre to centre.

The total depth of the section should be 24.3 inches. This will be more than realized when the corner at *F* is filled in as shown in Fig. 2. In no case should an abrupt change of direction be made in the profile at this section. Fillets should be built in at *F* and *G*.

The vertical shear at this section is about 14300 pounds per foot of wall. After the fillet is in place this is amply provided for.

The maximum stress near this section is probably that usually called diagonal tension, which might produce failure along an inclined surface starting near *F* and running downward toward the rear of the wall. This can be provided for by bending the ends of the horizontal reinforcing rods so that they cross diagonally from top to bottom below *FG*. See Fig. 2.

The projection of the foundation in the rear of the wall is treated in a manner similar to that employed above.

The moment at *G* is:

$$\begin{aligned} \text{Earth} & [26 \times 5 \times 100 = 13000] 2.5 = 32500 \\ \text{Concrete . . .} & [2 \times 5 \times 140 = 1400] 2.5 = 3500 \end{aligned} \} \text{ downward}$$

$$\begin{aligned} & [400 \times 5 = 2000] 2.5 = 5000 \\ & [333 \times 5 \times 0.5 = 833] \frac{1}{2} = 1390 \end{aligned} \} \text{ upward}$$

$$\text{Total} = 29600 \text{ ft.-lbs. per linear foot of wall'}$$

Using the same reinforcement employed in the forward portion, inspection shows that the section at *G* is ample to resist bending and shear.

Abutting Power.

Assuming that the earth in front of the wall and foundation well replaced, the total abutting value is (see page 43)

$$P = \frac{(4)^2}{2} 100 \times 3 = 2400 \text{ pounds per linear foot of wall.}$$

As this earth is almost entirely back-filled it will not be safe to depend upon it to resist very much sliding action of the wall. The tangent of the angle between the vertical and the resultant pressure upon the base is $\frac{12500}{20900} = 0.596$, corresponding to an angle of $30^\circ 49'$. This is greater than the angle of repose of the earth retained and hence some provision must be made to prevent the wall and foundation from sliding forward. One method to prevent sliding will be to build a wall in front of the foundation extending from the top to a point some 4 or 5 feet below the bottom of the foundation. This wall need not be over 2 feet thick (if reinforced with vertical bars) to safely resist shear and bending. As the wall can be placed so as to fill the excavation the abutting power of the earth in front will become available. Another method is to extend the wall below the foundation. This is effective, and furthermore provides secure anchorage for the vertical reinforcing bars, as shown in Fig. 2.

Anchorage of Bars.

The ends of the reinforcing bars must be extended beyond the sections sufficiently to develop the strength of the bars by the bond between concrete and steel.

The bars employed have a perimeter of $\frac{1}{4} \times 4 = 3.5$ inches, and hence the bond strength per linear inch of bar is $3.5 \times 80 = 280$ pounds. If 16,000 pounds per square inch is developed in the bar, it must be anchored for a length not less than $16000 \times 0.77 \div 280 = 44$ inches. This does not consider the effect of the deformed shape of the bars, which materially increases the bond. Since there is only 24 inches of concrete below *FG* without extending the wall downward, as explained above, the vertical bars can be anchored only by making this extension or by introducing a longitudinal bar or pipe to which the rods can be attached, as shown in Fig. 5.

Expansion.

Concrete has been assumed to have sensibly the coefficient of expansion of steel, but it is very questionable if any but a small range of temperature need be provided for. Designers of concrete walls have either provided vertical joints in the walls or introduced longitudinal bars near the face to prevent cracking under changes of temperature. Both methods have been successful.

If the area of the longitudinal bars is about $\frac{1}{60}$ of the area of the concrete profile, the effect of temperature changes will be provided for (very approximately).

Remarks.

Difficulties in construction will require a greater thickness at the top of the wall shown in Fig. 2. Also the back should be battered to a point below the frost line, as shown in Figs. 4, 5 and 6. As stated before, the junction of the wall proper with the base should be made gradually, as shown in Figs. 2, 3-6.

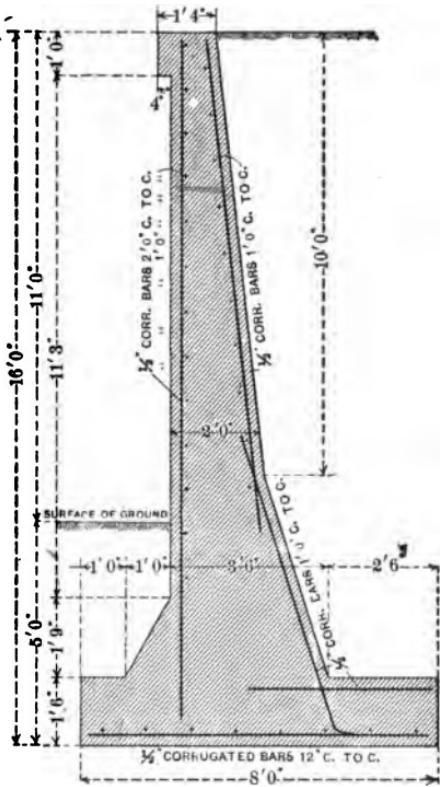


FIG. 3.

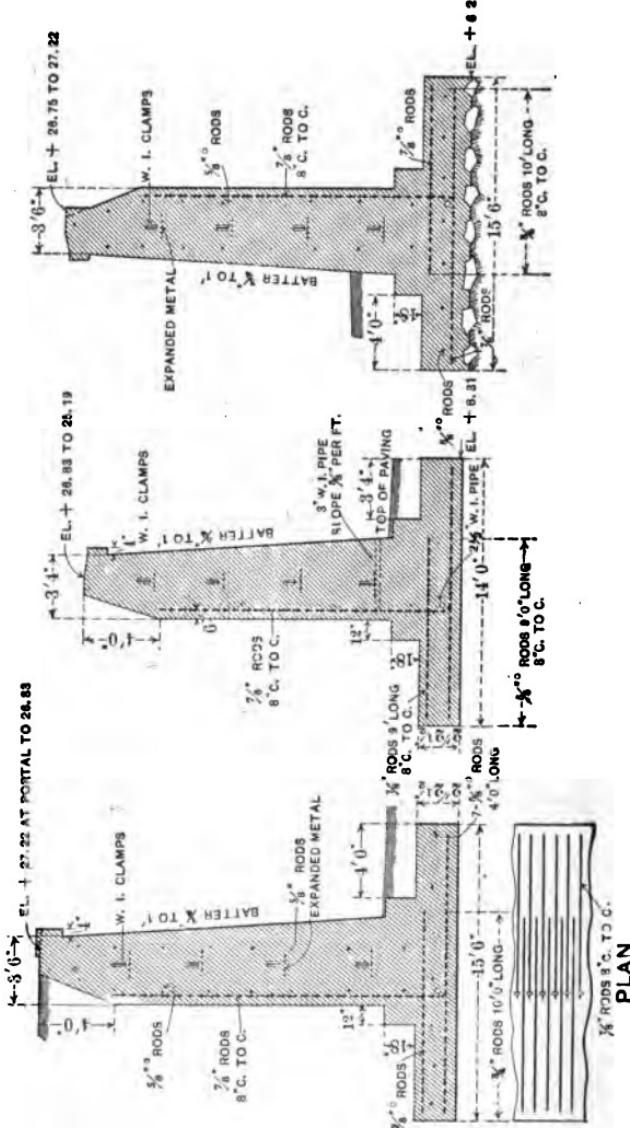


FIG. 6.

FIG. 5.

FIG. 4.

APPENDIX A.

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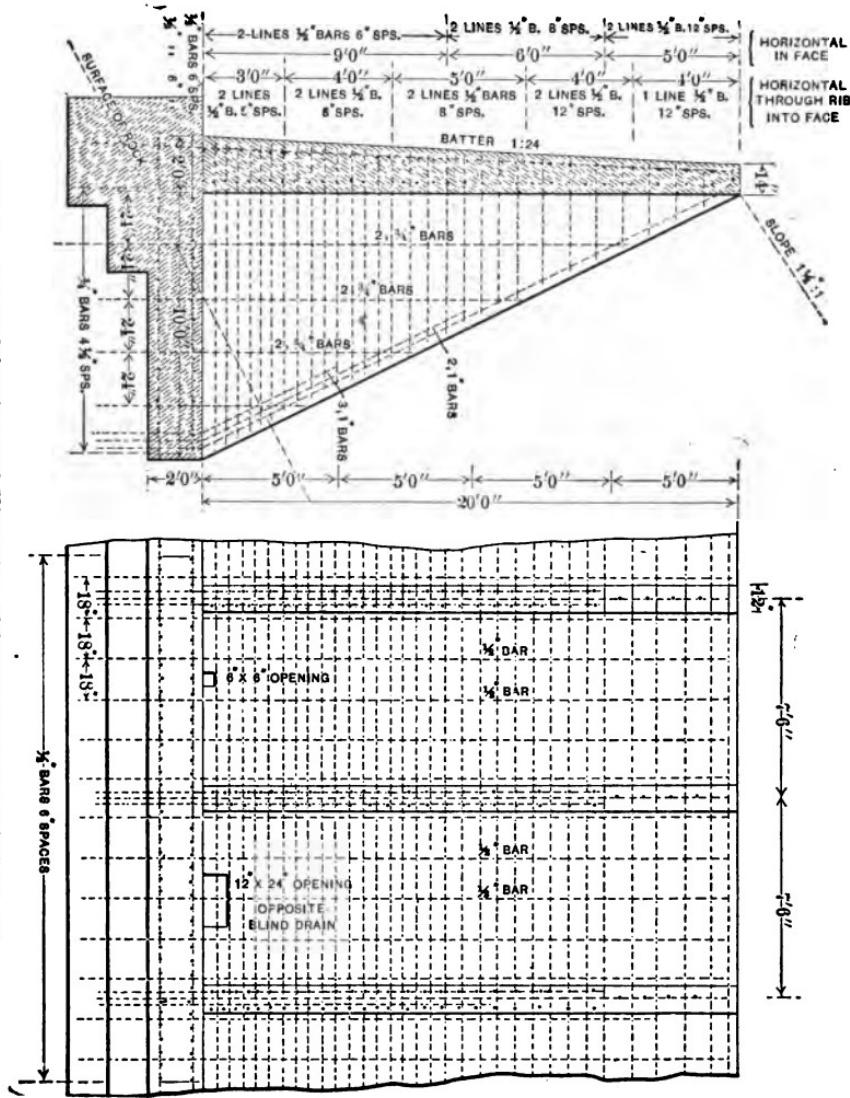


FIG. 7. - Typical Section and Elevation of Retaining-wall at Bridge 123.

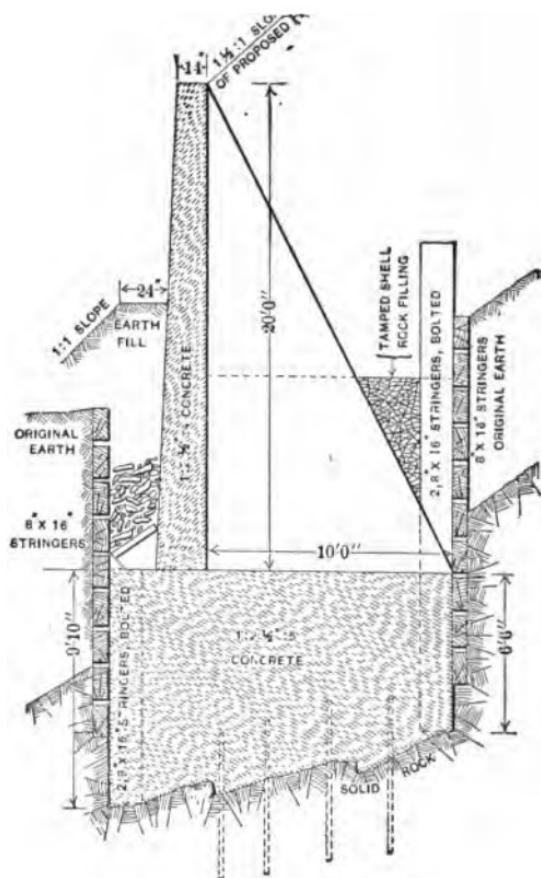


FIG. 8.—Section of Retaining-wall, Trench, and Timbering.

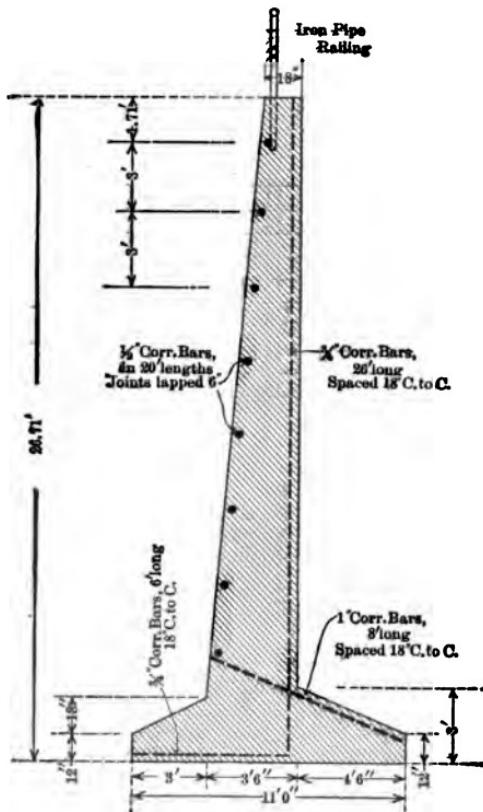


FIG. 9.—Section through the Dayton, O., Retaining Wall.

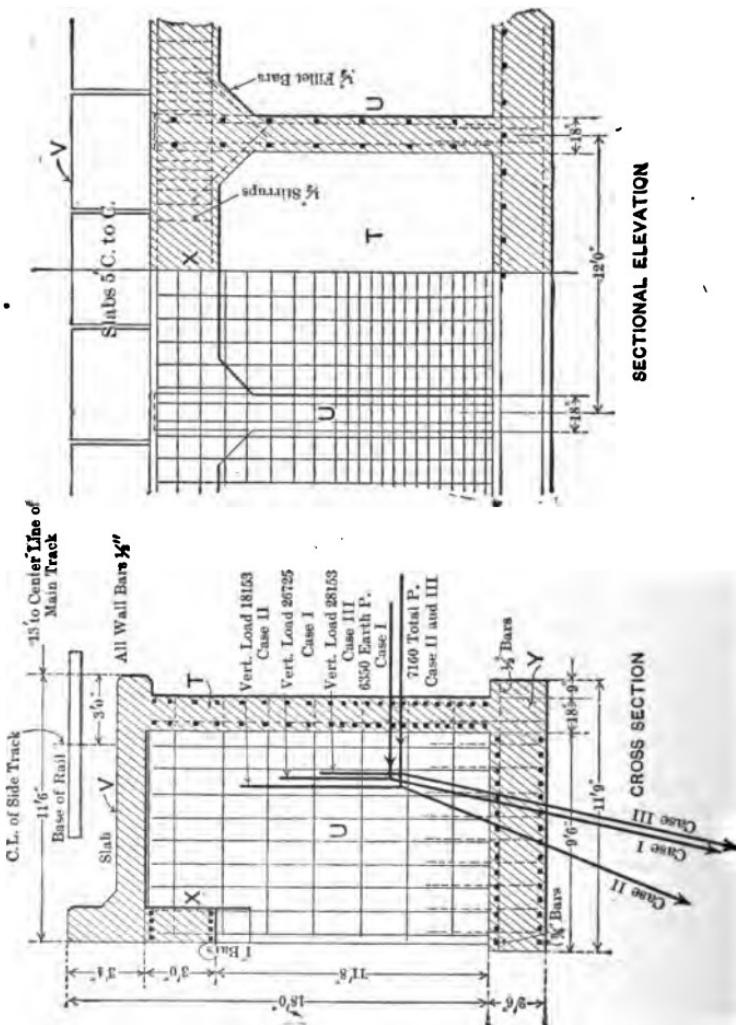


Fig. 10.—Reinforced-concrete Retaining Wall with Outside Buttresses and Slab Deck.

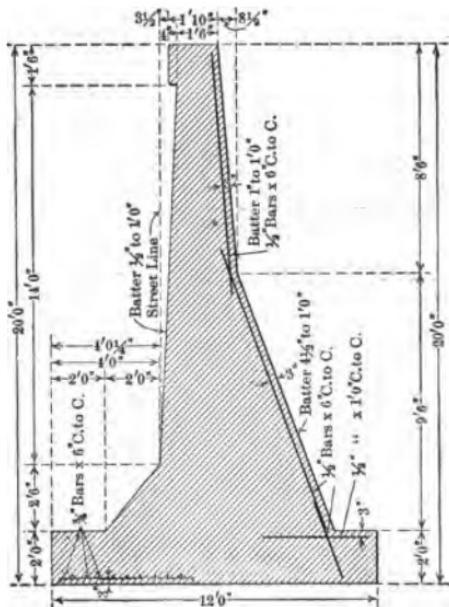


FIG. 11.—Typical Cross-section of Track Elevation Retaining Wall.

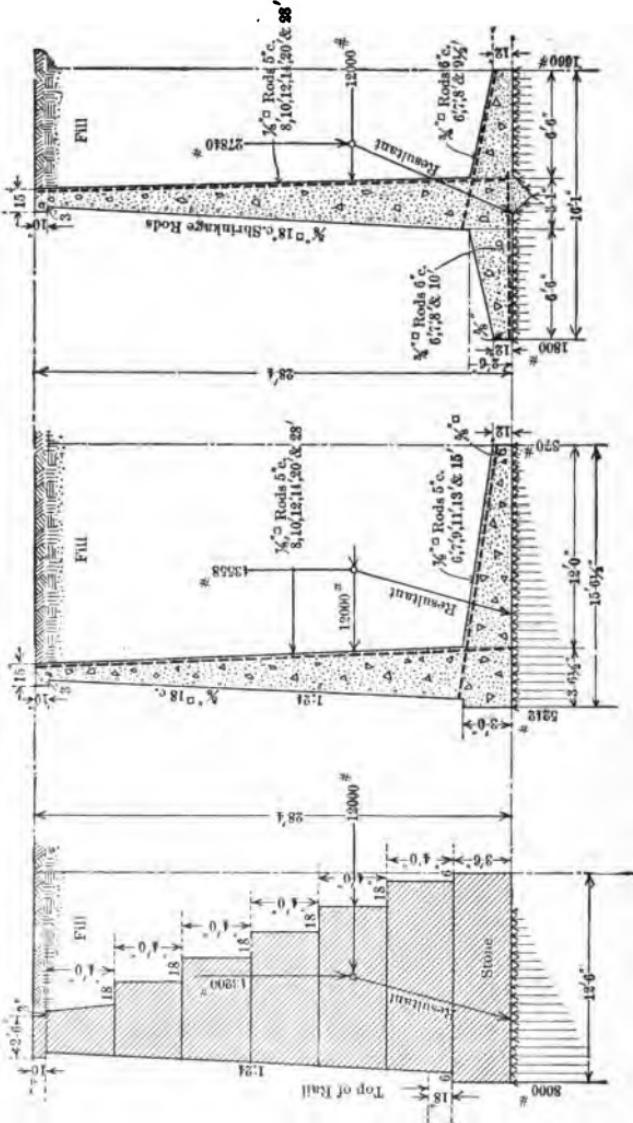


FIG. 13.
Three Types of Betainein, Walls
FIG. 14.

Fig. 12.

EXAMPLES FROM PRACTICE.

Fig. 3. A concrete wall at Indianapolis, Indiana, built in 1904 under the direction of Mr. H. W. Klausmann, County Engineer. The wall is 300 feet long without expansion joints, and after several months of winter weather showed no indication of cracking. (Strength of Reinforced Concrete, by T. L. Condron. Paper read March 15, 1905, Western Society of Engineers, Chicago.)

Figs. 4, 5, and 6. Examples of reinforced-concrete walls used in connection with the subway of the Philadelphia Rapid Transit Co. The vertical rods are bent around a 2½-inch pipe with a small radius, making an effective anchorage. (*The Engineering Record*, Feb. 25, 1905.)

Figs. 7 and 8. Walls constructed on The Great Northern Railway and described as follows by Mr. C. F. Graff, Locating Engineer C., M. & St. P. Ry., of Washington, in *Engineering News*, May 3, 1906:

"A reinforced-concrete wall of the type here described consists of three component parts, base, ribs, and face, all of which are so tied together by the embedded reinforcement as to assure monolithic action by the structure as a whole. The filling placed between the ribs and on top of the reinforced base assists in preventing overturning, and the saving of all the concrete thus replaced by filling is the principal cause of economy of this style of wall as contrasted with the ordinary gravity type of structure. The base and face may be assumed to act as continuous beams of equal span lengths, consisting of the distances centre to centre of ribs, in this case 7.5 feet. Such assumed action demands reinforcement near front and rear faces of face-wall and upper and lower faces of base. Referring to Fig. 7,

this horizontal reinforcement is indicated near top and bottom of base, a thickness of 2 feet having been assumed for the latter, but referring to Fig. 8 it is seen that the base as actually built possesses a far greater thickness, in some places no less than 12 feet, and it is clear that no horizontal reinforcement was required. Such was, therefore, omitted, this omission resulting in a saving of about \$600. All other reinforcement was placed exactly as shown in Fig. 7. The vertical reinforcing bars in the face-wall, Fig. 7, are inserted to prevent horizontal cracks in the face which would be induced by vertical stresses resulting from the slab action of the face-panel. For the same reason, and also to reinforce the toe of the wall, the transverse reinforcement is inserted in the base. The ribs or buttresses act as cantilevers in resisting the overturning moment of the earth filling which transmits its thrust from the face-wall to the ribs. The diagonal bars near the back faces of the ribs reinforce the latter against this cantilever action, and as this moment increases as the depth increases, the reinforcement section is increased correspondingly, as shown in Fig. 7, by the insertion of more bars. Vertical bars are placed in the ribs and assist the diagonals referred to in securely binding the ribs to the base. To prevent tensile failure between face-wall and rib the horizontal bars in the latter are inserted, such bars extending as near the face of the wall as possible. Shearing failure of the face near the ribs is also taken care of by the horizontal reinforcement in the former. This horizontal reinforcement is seen to increase as the depth increases to provide for increasing pressures. Johnson corrugated bars were used in this wall, and the Johnson formulas for average 1:3:6 rock concrete form the basis of design.

"Though the original design is based upon the assumption that the wall is free to tip around its toe, this condition by no means obtains. Fig. 8 makes it clear that the rock toe all along front of wall restrains this tipping action to a great extent. So also do the exceedingly massive base demanded and put in for other reasons referred to later, and the numerous anchor-bolts, binding the base to the rock foundation, assist in preventing tipping. It is probable, therefore, that the width of base consistently employed and equalling half the clear height of wall is rather excessive, but in view of the existing severe conditions and possible slides which would jar the structure, it was considered good practice to take every possible and practical precaution and pay less attention to the cost involved.

"Not for a great many years will the full pressures be brought upon this concrete wall on account of the permanent timbering behind same. Fig. 8 makes it apparent that this timbering will, so long as the same remains in a fair state of preservation, to a considerable extent relieve the pressures on the wall proper, and its presence will insure the attaining of a high strength by the concrete before it is put to the test."

Fig. 9. A wall at Dayton, Ohio. A portion of this wall failed by being undermined by water. *Engineering News*, March 10, 1910.

Fig. 10. A wall which was designed for the Chicago, Milwaukee and St. Paul Ry. under the condition that the wall should be on the property line but no part of the foundation to extend beyond this line. *Engineering News*, Apr. 20, 1911.

Fig. 11. A typical cross-section of track elevation

retaining wall built by the C., B. & Q. Ry. in Chicago.
Engineering World, March 8, 1907.

Figs. 12, 13 and 14. The three cross-seetions show
a gravity profile and two reinforced concrete profiles.
The renifored profiles were designed by Mr. F. A. Bone.
The Engineering Record, April 25, 1908.

APPENDIX B.

PROFILES OF GRAVITY WALLS RETAINING EARTH.

The following profiles were collected by the Committee on Masonry of the American Railway Engineering and Maintenance of Way Association. (Vol. 10, Part II, Proceedings, also *The Engineering Record*, Sept. 11, 1909.)

Fig. 1 illustrates a type of wall having considerable batter on the face and a nearly vertical back. For earth level with top of wall the resultant pressure on the base falls inside the middle third. For a surcharge of ten feet the resultant passes very near the toe.

Figs. 2-6 inclusive are profiles of walls retaining track elevation embankments. Several of these walls have moved forward at the top. Sections Figs. 4 and 5 are reported as not moving out; Fig. 6 has moved out $2\frac{1}{2}$ inches; Fig. 2, 4 inches; and Fig. 3, 11 inches.

Figs. 7 and 8 represent two old walls. Both have moved forward at the top, Fig. 8, 15 inches.

Fig. 9 is a profile of a wall which overturned and was replaced by Fig. 10, which has not moved (rock foundation).

Figs. 11 and 12 are walls without surcharge. Fig. 11 is concrete built in 1907 and has a pile foundation; no movement has been discovered. Fig. 12 is a rubble

masonry wall built 1897 and has moved out at the top 5 inches.

Fig. 13 is a track elevation wall that has moved out at the top $7\frac{1}{2}$ inches. The company that built this wall has adopted the profiles Figs. 14 and 15.

Figs. 16 and 17 are parallel walls for track elevation. Fig. 17 has moved out 4 inches at the top.

Figs. 18-21 inclusive indicate the profiles adopted by one railroad for track elevation.

Fig. 22 is a profile of a wall built by the Chicago and Northwestern Railway for track elevation in Chicago.

Another set of profiles of walls for retaining earth has been collected by Mr. Frank H. Carter, and published in *Engineering News* July 28, 1910. A number of these profiles are shown in Figs. 23-40 inclusive.

Fig. 23. N. Y., N. H. and H. R. R., Providence Division. Geo. T. Sampson, Div. Engineer.

Fig. 24. Penn., N. Y. and L. I. R. R. For water bearing earth. Contract drawings. Alfred Noble, Chief Engineer.

Fig. 25. Penn., N. Y. and L. I. R. R. For drained earth. Contract drawings. Alfred Noble, Chief Engineer.

Fig. 26. Boston Subway. Contract drawing. Sec. 11. H. A. Carson, Chief Engineer.

Fig. 27. East Boston Tunnel. Boston Transit Com. Sixth Report. H. A. Carson, Chief Engineer.

Fig. 28. Penn. Ave. Subway, Philadelphia, Pa. Geo. S. Webster, Chief Engineer.

Fig. 29. Detroit Tunnel. Concrete on pile foundation. Surcharged. Walls, Bins and Grain Elevators. Milo S. Ketchum.

Fig. 30. Borough of the Bronx, New York City.
S. C. Thompson, Prin. Asst. Engineer.

Fig. 31. Illinois Central R. R., Chicago, Ill. Built
in 1905. Concrete on pile foundation. Walls, Bins and
Grain elevators. Milo S. Ketchum.

Fig. 32. Boston and Maine R. R. Standard profile
for concrete. F. B. Rowell, Asst. Chief Engineer.

Fig. 33. Boston and Albany R. R. For embank-
ments without surcharge. When embankment is loaded
with a track assume a surcharge of six feet. William
Parker, Div. Engineer.

Fig. 34. Penn. R. R. Standard drawings. Wm. H.
Brown, Chief Engineer.

Fig. 35. N. Y. C. and H. R. R. Geo. W. Kittredge,
Chief Engineer.

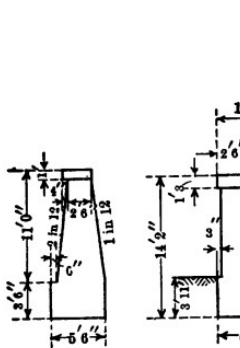
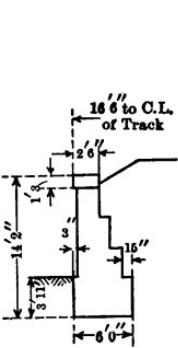
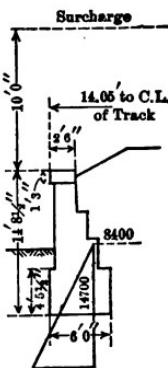
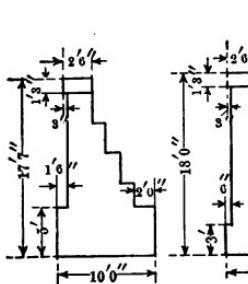
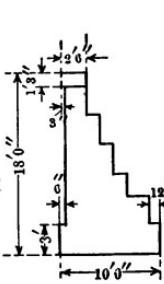
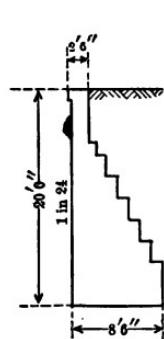
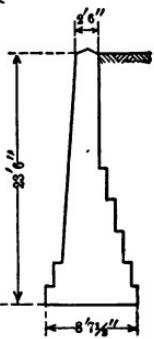
Fig. 36. Sea Wall. Lynn Shore Reservation, Met. Park
Commission. J. R. Roblin, Chief Engineer.

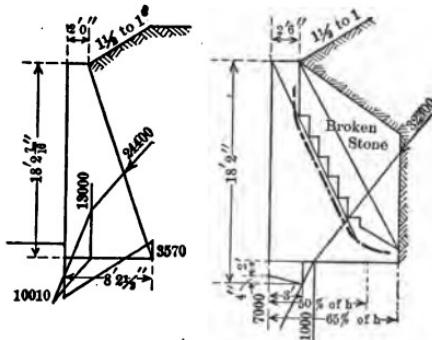
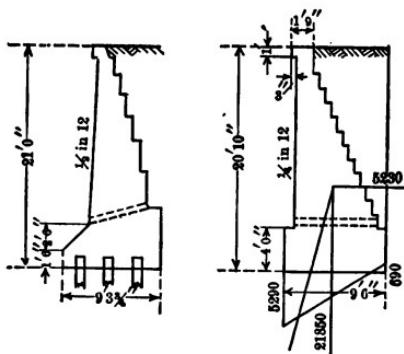
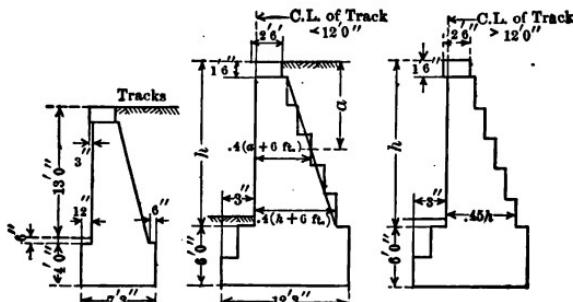
Fig. 37. Sea Wall. Cradock Bridge. Met. Park
Commission.

Fig. 38. Sea Wall, Charleston Bridge, Boston, Mass.
Wm. Jackson, City Engineer.

Fig. 39. Sea Wall on Charles River, City of Cambridge,
Mass. Park Dept. L. M. Hastings, City Engineer.

Fig. 40. Subway, South Station, Boston, Mass. Geo.
B. Francis, Chief Engineer.

FIG. 1.
($R=0.38$.)FIG. 2.
($R=0.43$.)FIG. 3.
($R=0.4$.)FIG. 4.
($F=0.57$.)FIG. 5.
($R=0.55$.)FIG. 6.
($R=0.46$.)FIG. 7. ($R=0.43$.)FIG. 8. ($R=0.37$.)

FIG. 9. ($R=0.45$). FIG. 10. ($R=0.65$)FIG. 11. ($R=0.44$). FIG. 12. ($R=0.46$)

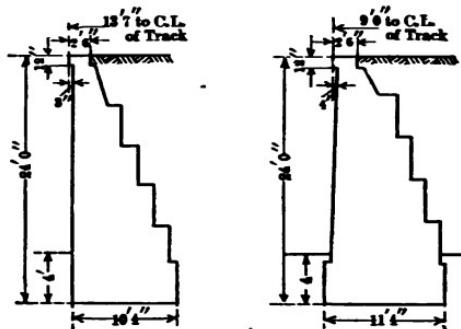
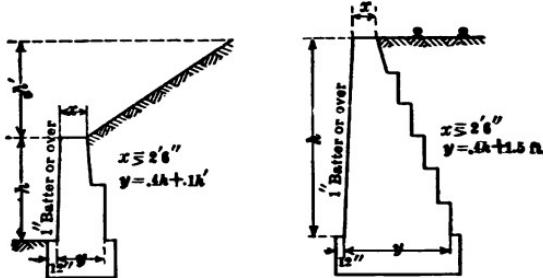
FIG. 16. ($R=0.43$)FIG. 17. ($R=0.47$)

FIG. 18.

FIG. 19.

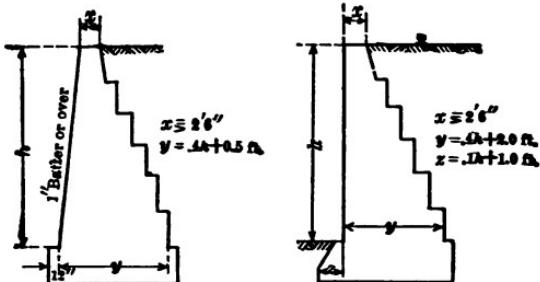


FIG. 20.

FIG. 21.

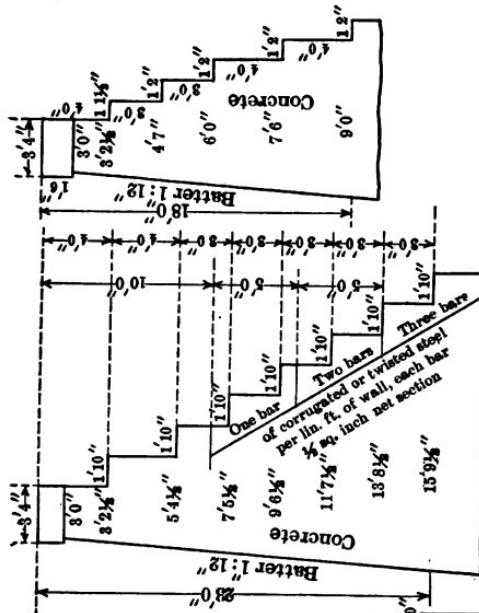


FIG. 25. (R=0.50.)

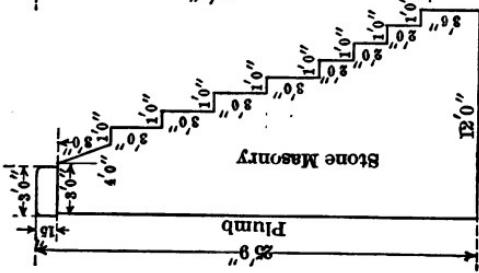


FIG. 24. (R=0.69.)

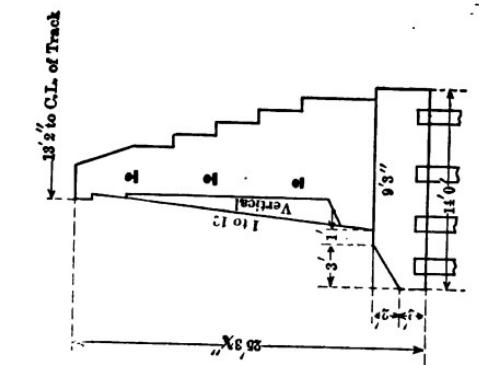


FIG. 23. (R=0.46.)

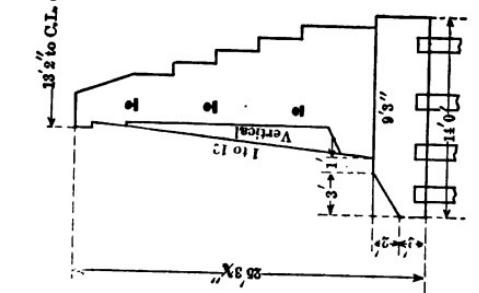


FIG. 22. (R=0.55.)

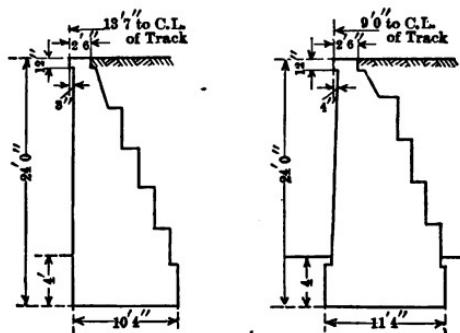
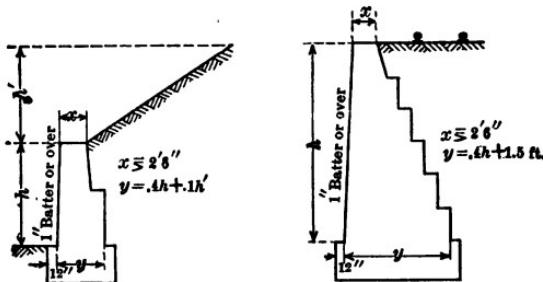
FIG. 16. ($R=0.48$.)FIG. 17. ($R=0.47$.)

FIG. 18.

FIG. 19.

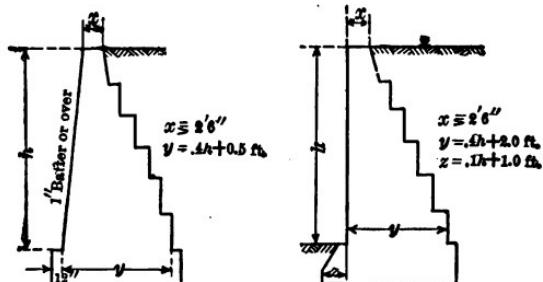


FIG. 20.

FIG. 21.

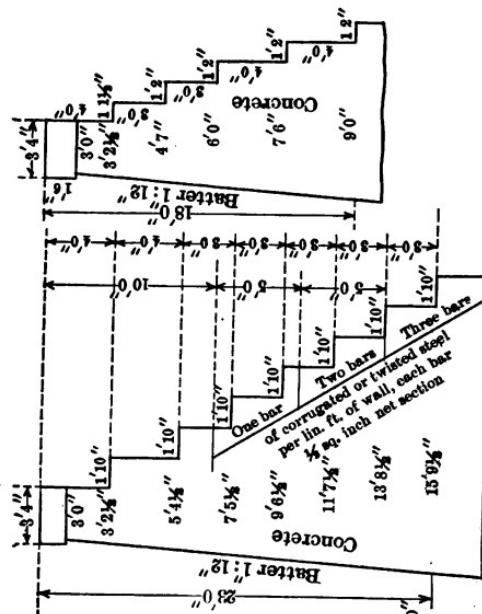


FIG. 25. (R=0.50.)

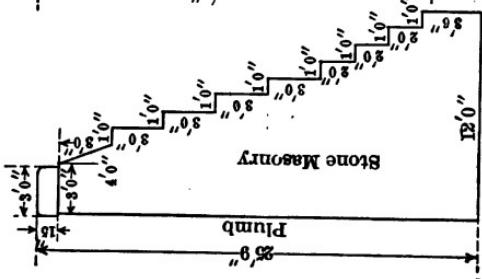


FIG. 24. (R=0.69.)

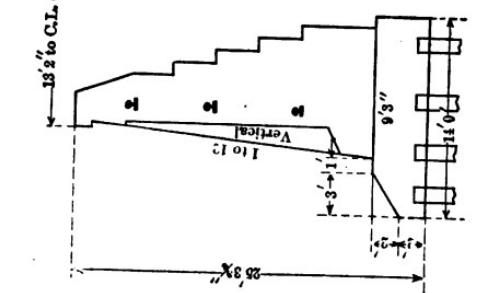


FIG. 23. (R=0.46.)

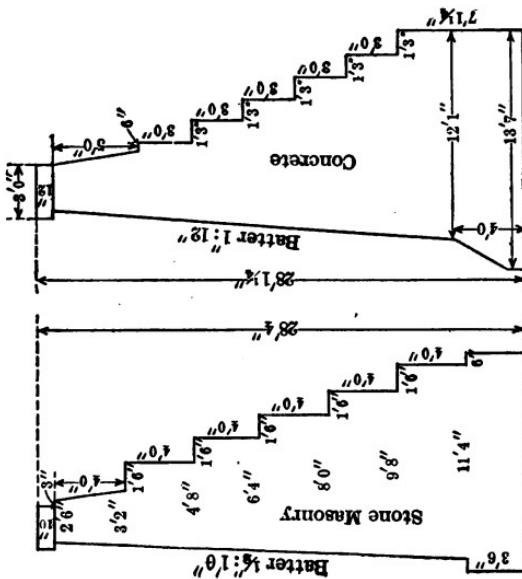


FIG. 29. ($R = 0.48$.)

FIG. 28. ($R = 0.43$.)

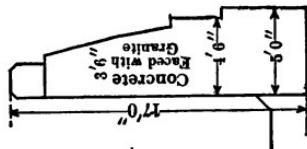


Fig. 27. ($R=0.30$.)

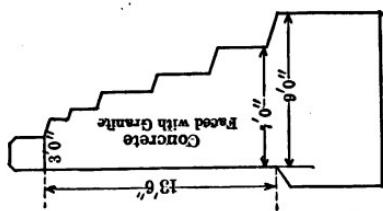


Fig. 26. ($R=0.52.$)
 ($R=0.66.$)

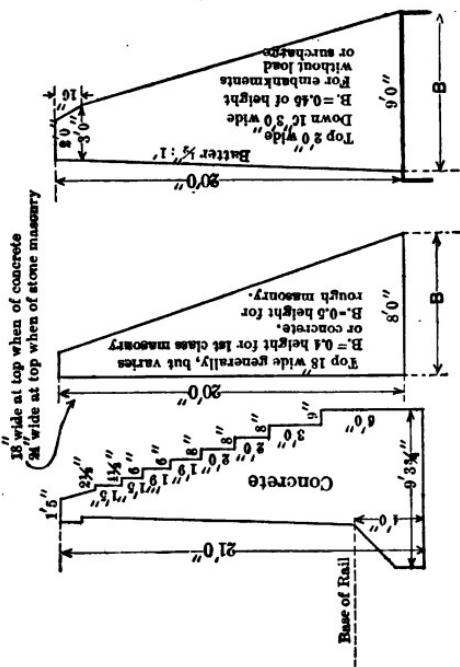


FIG. 31.
($R=0.44$.)

FIG. 32.
($R=0.40$.)

FIG. 33.
($R=0.45$.)

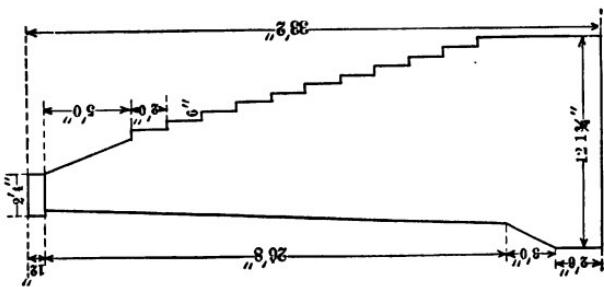
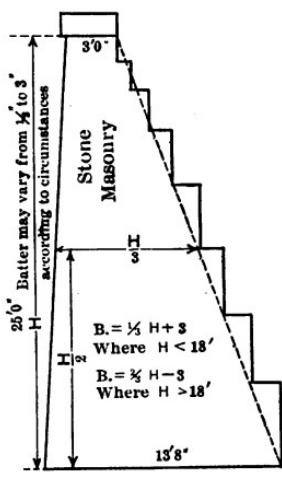
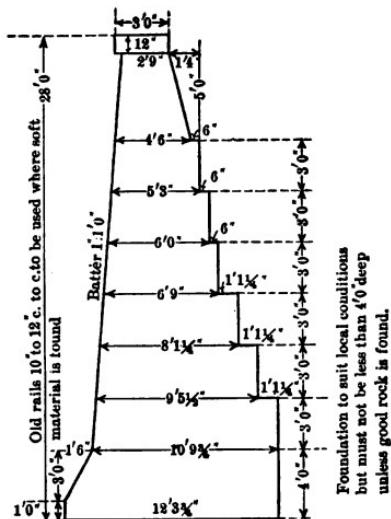
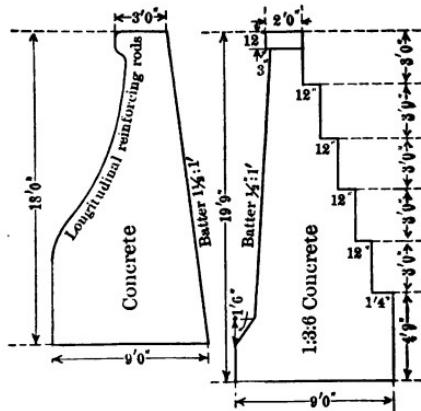
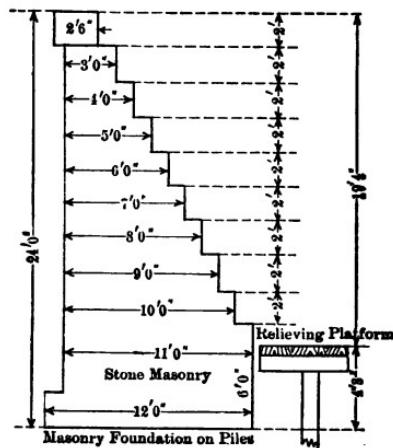
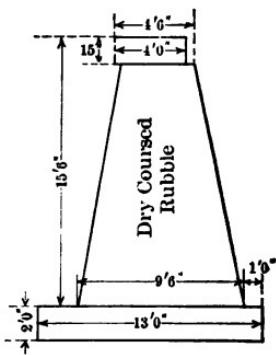
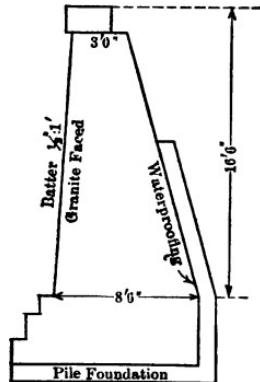
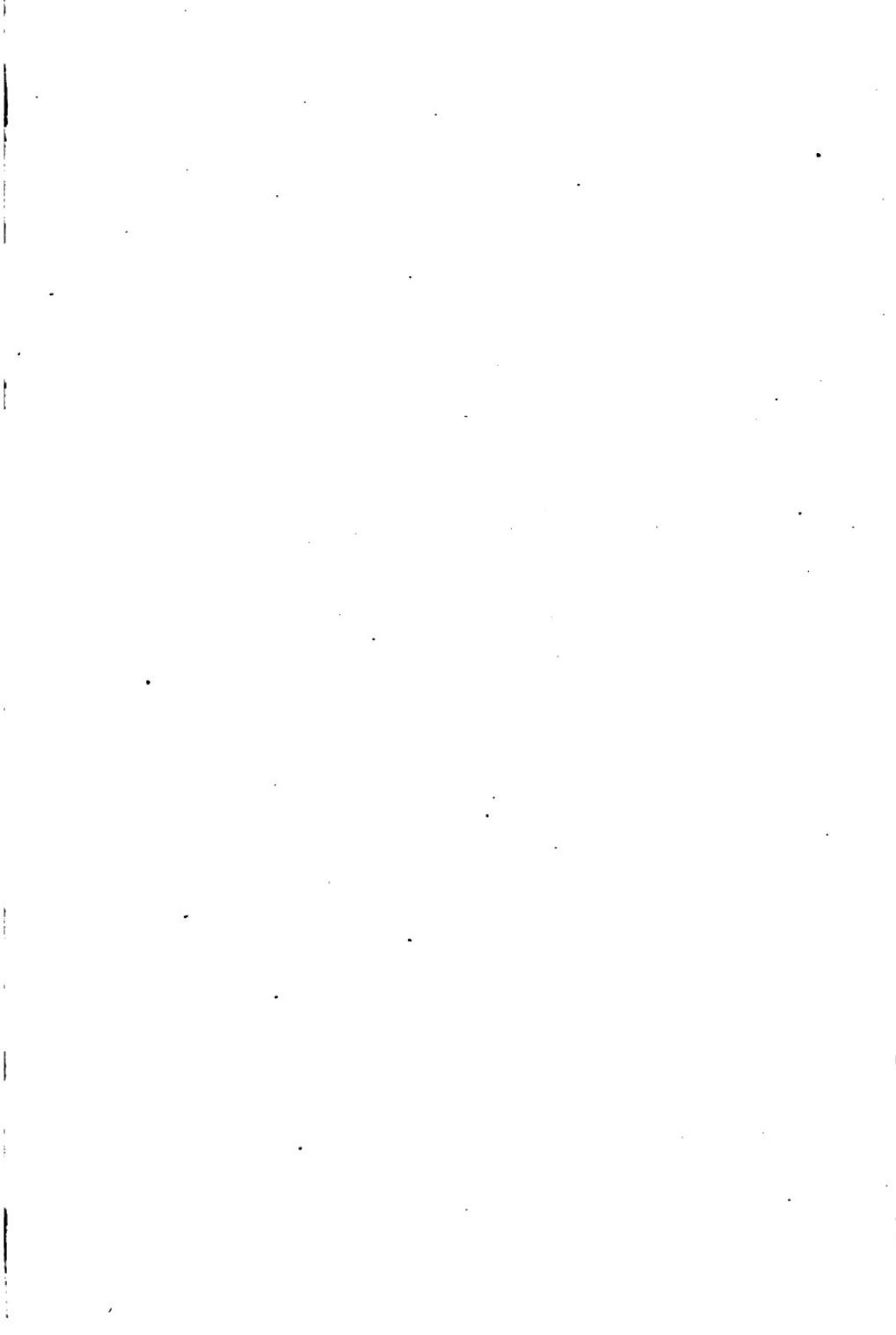


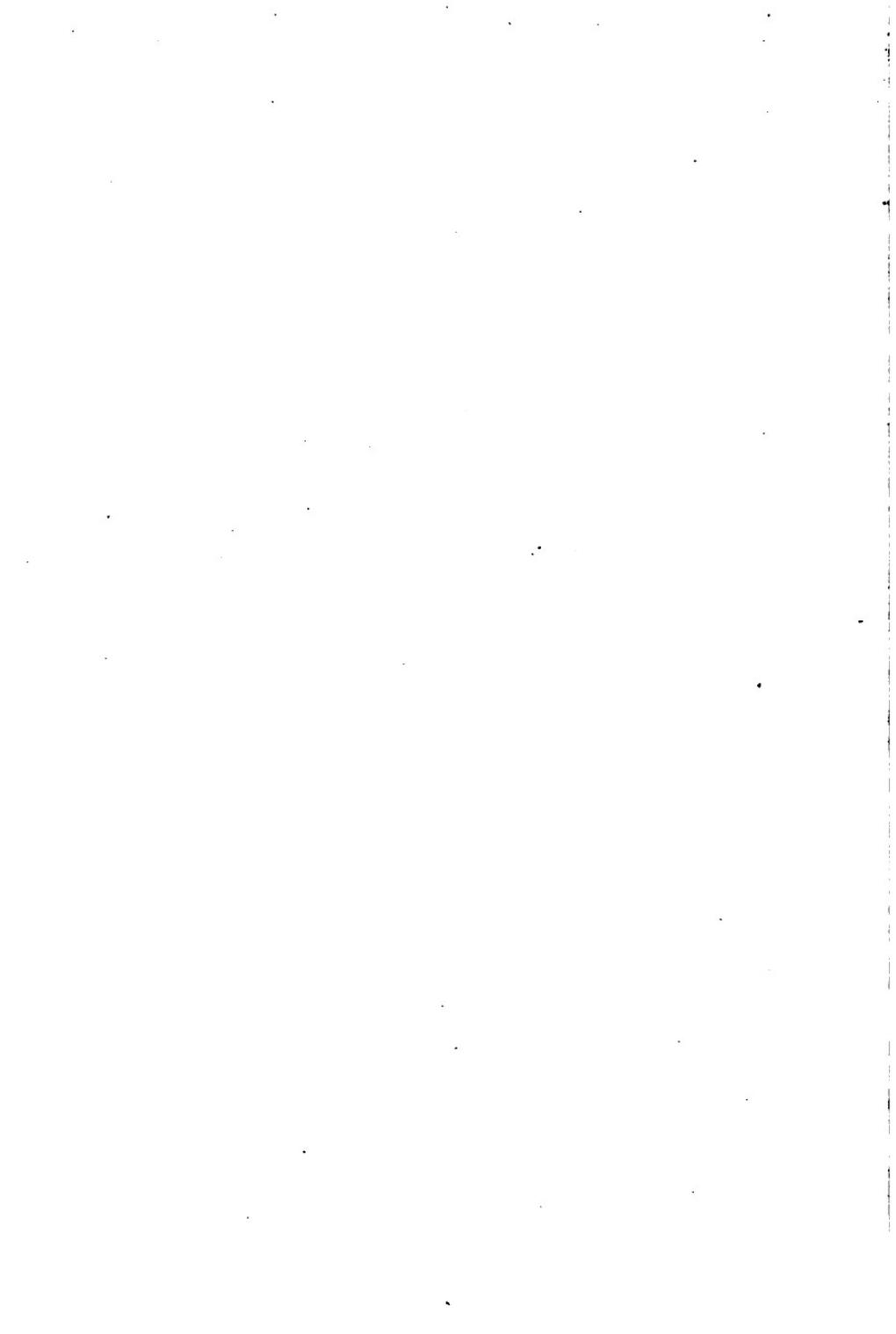
FIG. 30. ($R=0.36$.)

FIG. 34. ($R=0.55$.)FIG. 35. ($R=0.44$.)FIG. 36.
($R=0.50$.)FIG. 37.
($R=0.45$.)

FIG. 38. ($R=0.50$.)FIG. 39. ($R=0.61$.)FIG. 40. ($R=0.52$.)







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